

✿ Corrigé de l'examen Final ✿

Exercice 01 : (5.5 points)

1. $\mathbb{P}(R|M) = 0.5, \quad \mathbb{P}(R|\bar{M}) = 0.3 \quad \mathbb{P}(M) = 0.2$ **(1.5 points).**

2. De la première question on trouve : $\mathbb{P}(\bar{M}) = 1 - \mathbb{P}(M) = 0.8$.
 Donc

$$\begin{aligned} \mathbb{P}(R) &= \mathbb{P}(M) \cdot \mathbb{P}(R|M) + \mathbb{P}(\bar{M}) \cdot \mathbb{P}(R|\bar{M}) \\ &= 0.34 \quad \text{d'après la formule des probabilités totales} \quad \textbf{(2 points)} \end{aligned}$$

3.

$$\begin{aligned} \mathbb{P}(M|R) &= \frac{\mathbb{P}(M \cap R)}{\mathbb{P}(R)} \\ &= \frac{\mathbb{P}(R|M) \cdot \mathbb{P}(M)}{\mathbb{P}(R)} \\ &= 0.2941 \simeq 30\% \quad \textbf{(2 points)} \end{aligned}$$

Exercice 02 : (9 points)

1. Calcul de K :

$$\begin{aligned} f \text{ densité de probabilité} &\Rightarrow \int_0^4 Kx(4-x)dx = 1 \\ &\Rightarrow K = \frac{3}{32} \quad \textbf{(1 point)} \end{aligned}$$

2. La fonction de répartition :

$$F(x) = \begin{cases} 0 & \text{si } x < 0 \\ \frac{x^2}{16}(3 - \frac{x}{2}) & \text{si } 0 \leq x < 4 \\ 1 & \text{si } x \geq 4 \end{cases} \quad \textbf{(1.5 points)}$$

3. $\mathbb{P}(1 \leq X \leq 2) = F(2) - F(1) = \frac{11}{32} \simeq 0.2427$ **(1 point)**

4. $\mathbb{E}(X) = \frac{3}{32} \int_0^4 x^2(4-x)dx = 2$ **(1.5 point)**

5. $\mathbb{E}(X^2) = \frac{3}{32} \int_0^4 x^3(4-x)dx = \frac{24}{5} = 4.5$ **(1 point)**

D'où $\mathbb{V}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \frac{4}{5} = 0.8$ **(1 point)**

6. La loi de $Z = \sqrt{X}$:

$$f_Z(z) = \begin{cases} \frac{3}{16} z^3 (4 - z^2) & \text{si } z \in [0, 2] \\ 0 & \text{sinon} \end{cases} \quad (2 \text{ points})$$

Exercice 03 : (5.5 points)

1.

$$\begin{aligned} \mathbb{P}(X < 25000) &= \mathbb{P}\left(\frac{X - 35000}{5000} < \frac{25000 - 35000}{5000}\right) \\ &= \phi(-2) = 1 - \phi(2) = 0.02275 \quad (1.75 \text{ points}) \end{aligned}$$

2.

$$\begin{aligned} \mathbb{P}(25000 < X < 40000) &= \mathbb{P}\left(\frac{25000 - 35000}{5000} < \frac{X - 35000}{5000} < \frac{40000 - 35000}{5000}\right) \\ &= \phi(1) - \phi(-2) \\ &= 0.81859 \quad (2 \text{ points}) \end{aligned}$$

3.

$$\begin{aligned} \mathbb{P}(X > 45000) &= 1 - \mathbb{P}\left(\frac{X - 35000}{5000} < \frac{45000 - 35000}{5000}\right) \\ &= 1 - \phi(2) = 0.02275 \quad (1.75 \text{ points}) \end{aligned}$$