Univ Bouira 2019

Solution examen Vibrations et ondes

Questions de cours (6/6)

1) $\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = -\frac{\partial D}{\partial \dot{x}}$ (1)

2) $\lambda^2 - w_0^2 < 0 \Rightarrow$

$X(t) = A e^{-\lambda t}\cos\left(\sqrt{w_0^2-\lambda^2}\,t + \phi\right)$ (1)

$\lambda^2 - w_0^2 = 0 \Rightarrow$

$X(t) = e^{-\lambda t}(A_1 + A_2 t)$ (1)

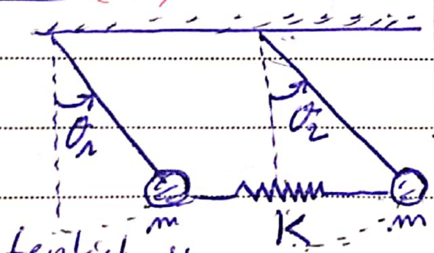
$\lambda^2 - w_0^2 > 0 \Rightarrow$

$X(t) = A_1 e^{(-\lambda - \sqrt{\lambda^2 - w_0^2})t} + A_2 e^{(-\lambda + \sqrt{\lambda^2-w_0^2})t}$ (1)

3) Equation Lagrange Syt amorty 2dl

$$\begin{cases} \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = -\frac{\partial D}{\partial \dot{x}_1} \\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} = -\frac{\partial D}{\partial \dot{x}_2} \end{cases}$$ (2)

Exercice 01 (6/6)



Energie potentiel U

$U = U_m + U_m + U_K$

$U = mg(l - l\cos\theta_1) + mg\,l(1-\cos\theta_2) + \frac{1}{2}Kl(\sin\theta_2 - \sin\theta_1)^2$

$\theta \ll \Rightarrow \cos\theta = 1 - \frac{\theta^2}{2}$ / $\sin\theta = \theta$

$U = \frac{1}{2}mgl\theta_1^2 + \frac{1}{2}mgl\theta_2^2 + \frac{1}{2}Kl^2(\theta_2-\theta_1)^2$ (0,5) (0,5) (0,5)

Energie cinétique : $T = T_m + T_m$

$T = \frac{1}{2}m l^2\dot{\theta}_1^2 + \frac{1}{2}m l^2 \dot{\theta}_2^2$ (0,5) (0,5)

3) **Equation differentielles du Système**

$$\begin{cases} \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0 \\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0 \end{cases}$$ (1)

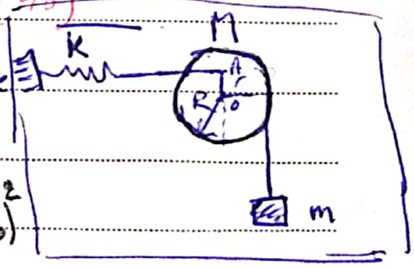
$$\begin{cases} ml^2\ddot{\theta}_1 + mgl\theta_1 + Kl^2(\theta_2-\theta_1)(-1) = 0 \\ ml^2\ddot{\theta}_2 + mgl\theta_2 + Kl^2(\theta_2 - \theta_1) = 0 \end{cases}$$

$$\begin{cases} ml^2\ddot{\theta}_1 + (mgl + l^2K)\theta_1 - Kl^2\theta_2 = 0 \\ ml^2\ddot{\theta}_2 + (lmg + l^2K)\theta_2 - Kl^2\theta_1 = 0 \end{cases}$$

$$\begin{cases} \ddot{\theta}_1 + \frac{(mgl + l^2K)}{ml^2}\theta_1 - \frac{Kl^2}{ml^2}\theta_2 = 0 \\ \ddot{\theta}_2 + \frac{(lmg + l^2K)}{ml^2}\theta_2 - \frac{Kl^2}{ml^2} = 0 \end{cases}$$

(2)
$$\begin{cases} \ddot{\theta}_1 + \left(\frac{mg + Kl}{ml}\right)\theta_1 - \frac{K}{m}\theta_2 = 0 \\ \ddot{\theta}_2 + \frac{(mg + Kl)}{ml}\theta_2 - \frac{K}{m}\theta_1 = 0 \end{cases}$$

Exercice 02 (8/8)



1/ **Energie potentielle**

$U = U_m + U_K$

$= mgh + \frac{K}{2}(x + x_0)^2$

$U = mgR\theta + \frac{1}{2}K(r\theta - x_0)^2$ (1)

$= mgR\theta + \frac{1}{2}K(r\theta)^2 + \frac{1}{2}Kx_0^2 - Kr\theta x_0$

2/ **Simplification de U**

$\left.\frac{\partial U}{\partial \theta}\right|_{\theta=0} = 0 \Rightarrow mgR - Krx_0 + Kr^2\theta\Big|_{\theta=0} = 0$ (1)

$\Rightarrow \boxed{mgR - Krx_0 = 0}$

on remplace dans U

$\Rightarrow U = \frac{1}{2}Kr^2\theta^2 + \underbrace{\frac{1}{2}Kx_0^2}_{ct} + \underbrace{(mgR - Krx_0)}_{0}\theta$

$\boxed{U = \frac{1}{2}Kr^2\theta^2 + ct}$ (1)

3) Energie cinétique T

$$T = T_m + T_M = \frac{1}{2} m (R\dot{\theta})^2 + \frac{1}{2}\left(\frac{1}{2} M R^2\right)\dot{\theta}^2$$

$$\boxed{T = \frac{1}{2}\left(m R^2 + \frac{1}{2} M R^2\right)\dot{\theta}^2}$$

(1)

4) Lagrangien et l'équation du mvt

$$\mathcal{L} = T - U = \frac{1}{2}\left(m R^2 + \frac{1}{2} M R^2\right)\dot{\theta}^2 - \frac{1}{2} K r^2 \theta^2 - c^{te}$$

(1)

Equation du mvt : $\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$

(1)

$$\Rightarrow \left(\frac{2mR^2 + MR^2}{2}\right)\ddot{\theta} + Kr^2\theta = 0 \Rightarrow \boxed{\ddot{\theta} + \frac{2Kr^2}{2mR^2 + MR^2}\theta = 0}$$

$$W_0 = \sqrt{\frac{2Kr^2}{2mR^2 + MR^2}} = 6{,}54 \text{ rad/s}$$

[illegible] $\Leftrightarrow$ AN ?

5) Equation horaire $\theta(t)$

$$\theta(t) = A \cos(\omega_0 t + \phi)$$

$$\theta(0) = A \cos(\phi) = 5 \Rightarrow \boxed{A = 5}$$

$$\dot{\theta}(0) = -A \sin(0) = 0$$

(2)

$$\boxed{\theta(t) = 5 \cos(6{,}54\, t)}$$