

Exo 3: $f(\theta) = \cos 2\theta + \cos^2 \theta$; $f: \mathbb{R} \rightarrow \mathbb{R}$

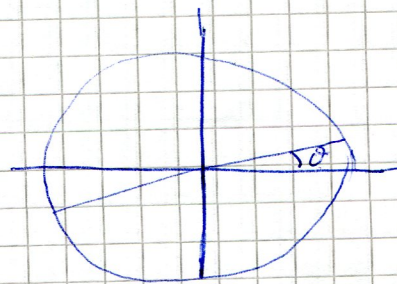
1).

• $f(\theta + 2\pi) = f(\theta) \leadsto D_1 = [-\pi, \pi]$

$f(\theta + \pi) = \cos(2\theta + 2\pi) + \cos^2(\theta + \pi)$

$= \cos 2\theta + (-\cos \theta)^2 = f(\theta)$

$\Rightarrow f$ est π p\'eriodique \Rightarrow sym $^\circ$ \`a l'origine $\leadsto D_1 = [-\frac{\pi}{2}, \frac{\pi}{2}]$



2) • $f(-\theta) = \cos(-2\theta) + \cos^2(-\theta)$

$= \cos(2\theta) + \cos^2 \theta = f(\theta)$

$\Rightarrow f$ paire $\leadsto D_2 = [0, \frac{\pi}{2}]$

\Rightarrow sym $^\circ$ (x x') $\leadsto D_2 = [0, \frac{\pi}{2}]$

3) $f(\theta_0) = 0 \Rightarrow \cos 2\theta_0 = -\cos^2 \theta_0$

$\Rightarrow \cos^2 \theta_0 = -\cos 2\theta_0 > 0 \Rightarrow \theta_0 \in]-\frac{\pi}{2}, \frac{\pi}{2}[$

$\theta_0 \in [0, \frac{\pi}{2}] \Rightarrow 2\theta_0 \in [0, \pi]$

$-\cos 2\theta_0 > 0 \Rightarrow 2\theta_0 \in [\frac{\pi}{2}, \pi] \Rightarrow \theta_0 \in [\frac{\pi}{4}, \frac{\pi}{2}]$

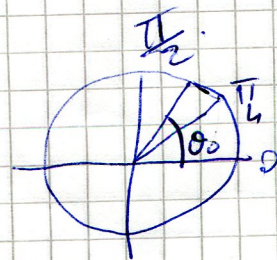
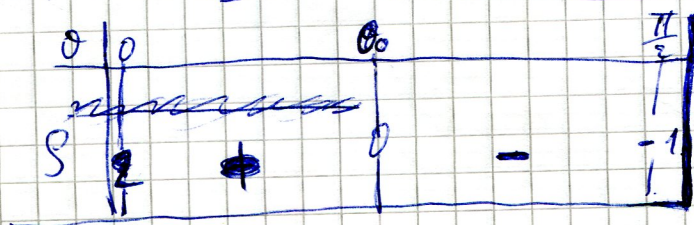
$\Rightarrow \cos \theta_0 > 0 \Rightarrow \cos \theta_0 = \sqrt{\cos 2\theta_0}$

$\cos 2\theta_0 = \cos^2 \theta_0 - \sin^2 \theta_0 = \cos^2 \theta_0 - (1 - \cos^2 \theta_0)$
 $= 2\cos^2 \theta_0 - 1$

$\Rightarrow \cos^2 \theta_0 = 2\cos^2 \theta_0 - 1 \Rightarrow \cos^2 \theta_0 = \frac{1}{3}$

~~$\cos \theta_0 \in [\frac{\pi}{4}, \frac{\pi}{2}] \Rightarrow \theta_0 \in \frac{\pi}{4}$~~ ($\cos^2 \theta_0 = \frac{1}{3} \Rightarrow \cos \theta_0 = \frac{\sqrt{3}}{3}$)

$\Rightarrow \cos \theta_0 = \frac{1}{\sqrt{3}} \Rightarrow \theta_0 = \arccos \frac{1}{\sqrt{3}}$



$f(\theta) > 0 \Rightarrow \cos^2 \theta > -\cos 2\theta$

$\Rightarrow \cos^2 \theta > -(2\cos^2 \theta - 1)$

$\Rightarrow \cos^2 \theta > \frac{1}{3} \Rightarrow \cos \theta > \frac{1}{\sqrt{3}}$

$\Rightarrow \theta < \theta_0$