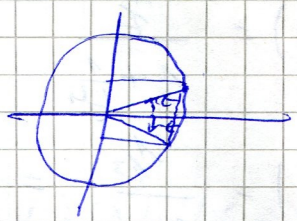
Exo 2 : $F_1 : \mathbb{R} \longrightarrow \mathbb{R}^2$

$t \longrightarrow (\cos^3 t, \sin^3 t) = (x(t), y(t))$

$\mathcal{D}_{F_1} = \mathbb{R}$.

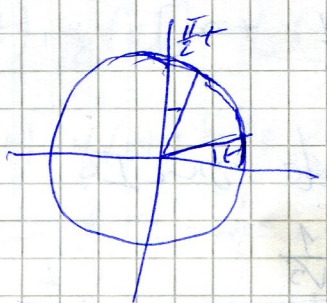
x, y 2π-périodiques $\rightsquigarrow \mathcal{D}_1 = [-\pi, \pi]$.

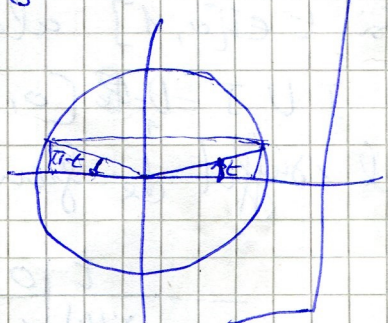
$y(-t) = -y(t)$
$x(-t) = x(t)$ $\rightsquigarrow \mathcal{D}_2 = [0, \pi]$.
$\rightsquigarrow$ sym./ $0x$.

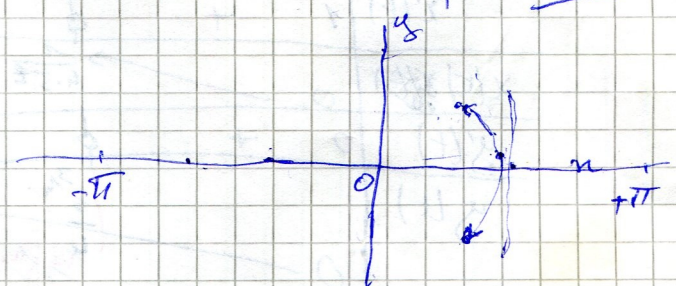


$x(\pi - t) = -x(t)$
$y(\pi - t) = y(t)$ $\rightsquigarrow \mathcal{D}_3 = [0, \frac{\pi}{2}] \rightsquigarrow$ sym./ $0y$

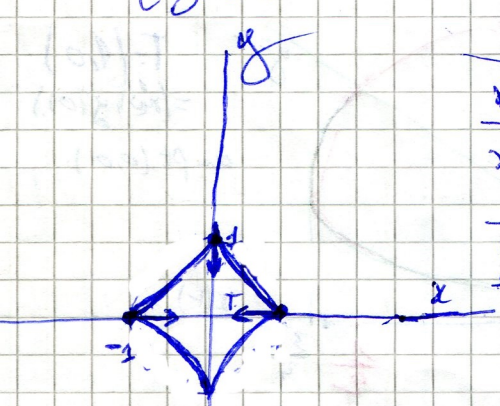
$x(\frac{\pi}{2} - t) = y(t)$
$y(\frac{\pi}{2} - t) = x(t)$ $\rightsquigarrow \mathcal{D}_3 = [0, \frac{\pi}{4}] \rightsquigarrow$ sym./ 1ère bissec







$\begin{cases} x'(t) = -3 \sin^2 t \cos^2 t \\ y'(t) = 3 \cos^2 t \sin^2 t \end{cases}$



t		$\frac{\pi}{2}$		π
$x'(t)$	0		0	
$x(t)$	1		$\searrow -1$	
$y'(t)$	0	$+$	0	$-$ 0
$y(t)$	0	$\nearrow 1$		$\searrow 0$

$(1,0)$ et $(-1,0)$ pts stationnaires : $(x'(t) = y'(t) = 0)$

$\begin{cases} x''(t) = -3\cos^3 t + 6 \sin t \cos t \\ y''(t) = -3 \sin^3 t + 6 \cos^2 t \sin t \end{cases} \rightarrow \begin{cases} x''(0) = -3 \\ y''(0) = 0 \end{cases}$; $\begin{cases} x''(\pi) = \\ y''(\pi) = \end{cases}$

$\Rightarrow T_{(1,0)} = \langle (-3, 0) \rangle$; ~~raisonnement pour $t = \pi$~~