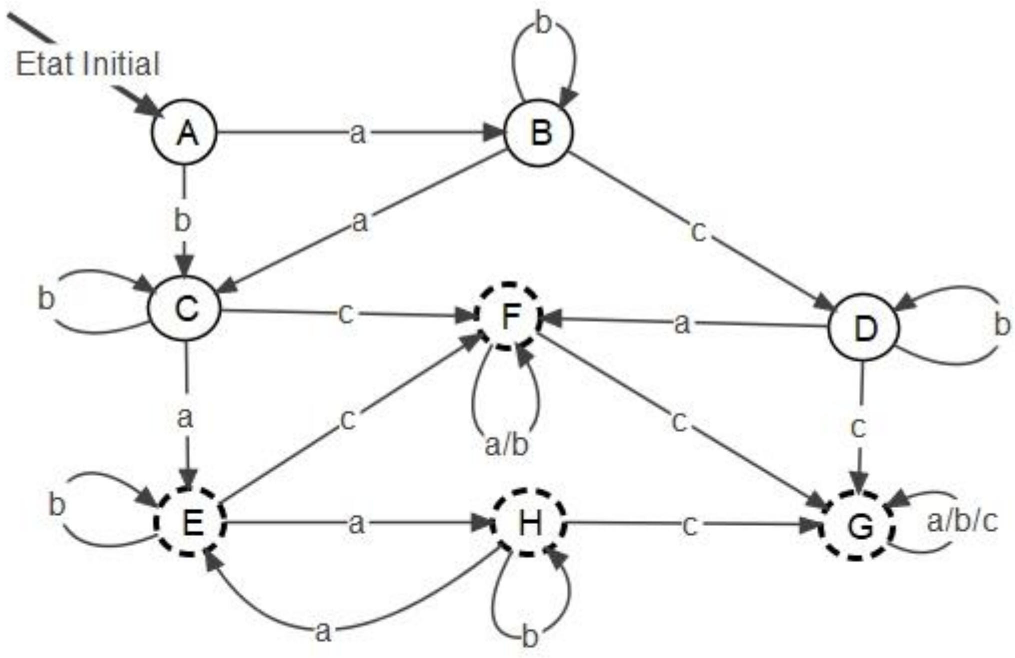
Solution

<u>Exercice 1</u> : (8 points)

1.

Q \ X	a	b	c
0 (A)	1 (B)	0, 2 (C)	-
1 (B)	0, 2 (C)	1 (B)	2 (D)
0, 2 (C)	1, 2, 4 (E)	0, 2 (C)	4 (F)
2 (D)	2, 4 (F)	2 (D)	4 (G)
1, 2, 4 (E)	0, 2, 4 (H)	1, 2, 4 (E)	2, 4 (F)
2, 4 (F)	2, 4 (F)	2, 4 (F)	4 (G)
4 (G)	4 (G)	4 (G)	4 (G)
0, 2, 4 (H)	1, 2, 4 (E)	0, 2, 4 (H)	4 (G)

Etat initial : A.
Etats finaux : E, F, G, H.



2. C_1 = {A, B, C, D} ; C_2 = {E, F, G, H}.

Testons la classe C_1 :

A // a = B € C_1 A // b = C € C_1 A // c = Ø
B // a = C € C_1 B // b = B € C_1 B // c = D € C_1
C // a = E € C_2 C // b = C € C_1 C // c = F € C_2
D // a = F € C_2 D // b = D € C_1 D // c = G € C_2

Séparer les deux classes {A} et {B}.

$C_1 = \{C, D\}$; $C_2 = \{E, F, G, H\}$; $C_3 = \{A\}$; $C_4 = \{B\}$.

Testons la classe C_1 (à nouveau après avoir fait la séparation) :

C // a = E Є C_2	C // b = C Є C_1	C // c = F Є C_2
D // a = F Є C_2	D // b = D Є C_1	D // c = G Є C_2

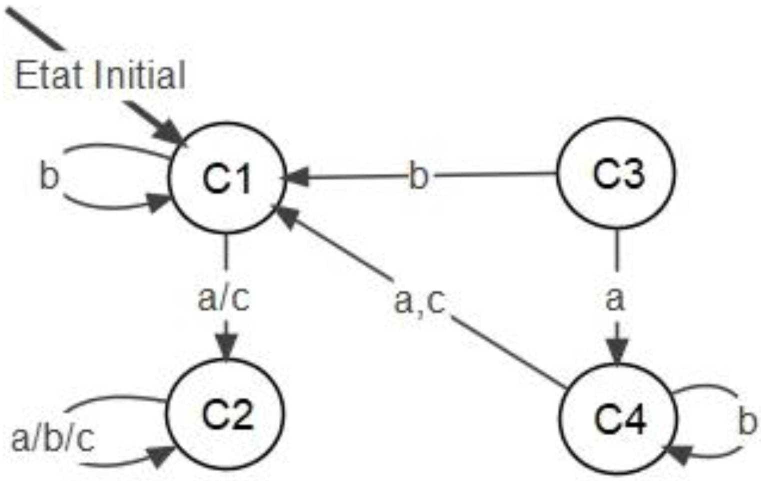
Pas de séparation.

Testons la classe C_2 :

E // a = H Є C_2	E // b = E Є C_2	E // c = F Є C_2
F // a = F Є C_2	F // b = F Є C_2	F // c = G Є C_2
G // a = G Є C_2	G // b = G Є C_2	G // c = G Є C_2
H // a = E Є C_2	H // b = H Є C_2	H // c = G Є C_2

Pas de séparation.

$C_1 = \{C, D\}$; $C_2 = \{E, F, G, H\}$; $C_3 = \{A\}$; $C_4 = \{B\}$.



Exercice 2 : (6 points)

Soit le langage suivant **L = a(b + ab)* + b⁺(a* + ab⁺a)**

1. $q_0 = L = a(b + ab)^* + b^+(a^* + ab^+a)$ Є Q_f.

   q_0 // a = $(b + ab)^* = q_1$ Є Q_f.
   q_0 // b = $b^*(a^* + ab^+a) = q_2$ Є Q_f.

   q_1 // a = $b(b + ab)^* = q_3$.
   q_1 // b = $(b + ab)^* = q_1$ Є Q_f.

   q_2 // a = $a^* + b^+a = q_4$ Є Q_f.
   q_2 // b = $b^*(a^* + ab^+a) = q_2$ Є Q_f.

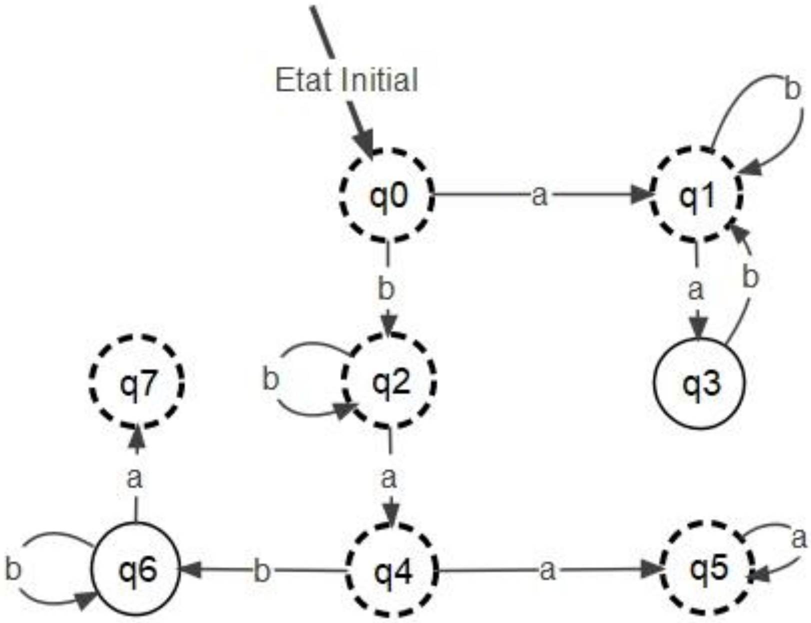
   q_3 // a = Ø.
   q_3 // b = $(b + ab)^* = q_1$ Є Q_f.

q_4 // a = a^* = q_5 Є Q_f.
q_4 // b = b^*a = q_6.

q_5 // a = a^* = q_5 Є Q_f.
q_5 // b = Ø.

q_6 // a = Ɛ = q_7 Є Q_f.
q_6 // b = b^*a = q_6.

q_7 // a = Ɛ = Ø.
q_7 // b = Ɛ = Ø.



2. $\delta^*(q_0$, ababa) |-- (a, q_1, baba) |-- (ab, q_1, aba) |-- (aba, q_3, ba) |-- (abab, q_1, a)
   |-- (ababa, q_1, Ɛ).
   Vu que « q_1 » n'est pas un état final, le mot ababa n'appartient pas au langage L.

   $\delta^*(q_0$, baba) |-- (b, q_2, aba) |-- (ba, q_4, ba) |-- (bab, q_6, a) |-- (abab, q_7, Ɛ).
   Vu que « q_7 » est un état final, le mot baba appartient au langage L.

<u>Exercice 3</u> : (6 points)

1. $\delta(q_0$, a, #) = (q_1, a#).
   $\delta(q_1$, a, a) = (q_1, aa).
   $\delta(q_1$, b, a) = (q_2, Ɛ).
   $\delta(q_1$, c, a) = (q_3, a).
   $\delta(q_2$, b, a) = (q_2, Ɛ).
   $\delta(q_2$, c, a) = (q_3, a).
   $\delta(q_2$, c, #) = (q_6, #).
   $\delta(q_3$, c, a) = (q_4, a).

$\delta(q_4, c, a) = (q_5, \varepsilon)$.
$\delta(q_5, c, a) = (q_3, a)$.
$\delta(q_5, c, \#) = (q_6, \#)$.
$\delta(q_6, c, \#) = (q_7, \#)$.
$\delta(q_7, c, \#) = (q_8, \#)$.
$\delta(q_8, \varepsilon, \#) = (q_f, \#)$.

2. $\delta^*(q_0, abccc, \#) \vdash (q_1, bccc, a\#) \vdash (q_2, ccc, \#) \vdash (q_6, cc, \#)$
$\vdash (q_7, c, \#) \vdash (q_8, \varepsilon, \#) \vdash (q_f, \varepsilon)$.
Le mot abccc appartient au langage L.