TD sur les guides rectangulaires

Exo 4

Dimensions $2{,}5 \times 1\ cm^2 \Rightarrow a = 2{,}5\ cm,\ b = 1\ cm.$

fréquence de propagation $f_0 = 8{,}6\ GHz$

- Les différents modes qui peuvent se propager sont :

$TE_{10},\ TE_{01},\ TE_{11},\ TE_{21},\ TE_{12},\ TE_{22},\ \ldots$

$TM_{11},\ TM_{21},\ TM_{12},\ TM_{22}\ \ldots$

- Mode $TE_{10} \Rightarrow m = 1,\ n = 0$ la fréquence de coupure f_c s'écrit :

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + 0} = \frac{c}{2\cancel{\pi}} \times \frac{\cancel{\pi}}{a} = \frac{3.10^8}{2 \times 2{,}5.10^{-2}} = 6\ GHz.$$

$f_0 > f_c \Rightarrow$ ce mode est possible.

Mode $TE_{01} \Rightarrow m = 0,\ n = 1 \rightarrow f_c = \frac{c}{2\pi}\sqrt{0 + \left(\frac{\pi}{b}\right)^2} = \frac{3.10^8}{2.10^{-2}} = 1{,}5.10^{10}$

$\boxed{f_c = 1{,}5.10^{10}\ Hz}$; $f_0 < f_c$, TE_{01} ne peut pas se propager dans le guide.

- $TE_{11} \rightarrow m = 1,\ n = 1 \Rightarrow f_c = \frac{c}{2\pi}\sqrt{\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}} = \frac{c}{2}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)$

$$f_c = \frac{3.10^8}{2}\left(\frac{1}{(2{,}5.10^{-2})^2} + \frac{1}{(10^{-2})^2}\right) = 1{,}74.10^{12}\ Hz.$$

$f_0 < f_c$ ce mode ne peut pas se propager.

$\lambda_g = ?$ longueur d'onde du guide.

$\gamma^2 + \omega^2 \varepsilon_0 \mu_0 = k^2$ avec $k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$\gamma^2 + \omega^2 \varepsilon_0 \mu_0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \Rightarrow \gamma^2 = -\omega^2 \varepsilon_0 \mu_0 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

Sachant que $\gamma = j k_g \Rightarrow \gamma^2 = -k_g^2$

$-k_g^2 = -\omega^2 \varepsilon_0 \mu_0 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \Rightarrow k_g^2 = -\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] + \omega^2 \varepsilon_0 \mu_0$

$$k_g = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \omega^2 \varepsilon_0 \mu_0} = \sqrt{\omega^2 \varepsilon_0 \mu_0 - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$k_g = \sqrt{\omega^2 \varepsilon_0 \mu_0 - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \Rightarrow \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \frac{2\pi f_c}{c}$$

$$\frac{\omega_c}{c} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \Rightarrow \frac{\omega_c^2}{c^2} = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$k_g = \sqrt{\frac{\omega_0^2}{c^2} - \frac{\omega_c^2}{c^2}} = \frac{\omega_0}{c}\sqrt{1 - \left(\frac{\omega_c}{\omega_0}\right)^2}$$

avec $\omega_0^2 \varepsilon_0 \mu_0 = \omega_0^2 / c^2$

$$k_g = 2\pi \frac{f_0}{c}\sqrt{1 - \left(\frac{f_c}{f_0}\right)^2} \Rightarrow \frac{\cancel{2\pi}}{\lambda_g} = \frac{\cancel{2\pi}}{\lambda_0}\sqrt{1 - \left(\frac{f_c}{f_0}\right)^2}$$

avec $\lambda_0 = \dfrac{c}{f_0}$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{f_c}{f_0}\right)^2}} \qquad \lambda_0 = \frac{3.10^8}{8,6.10^9} = 3,488\,m$$

- ce qui donne $\lambda_g = \dfrac{3,488}{\sqrt{1 - \left(\dfrac{6.10^9}{8,6.10^9}\right)^2}} = 4,868\,m$

- qui correspond au mode TE_{10}

Exercice N° 2. $a = 2\,cm$, $b = 1\,cm$.

$$H_z = H_0 \cos\frac{m\pi}{a}\cos\frac{n\pi}{b}\, e^{j(\omega t - k_g z)}$$

. TE mode $\Rightarrow E_z = 0$, $H_z \neq 0$, E est transverse.

$$\Delta H + \varepsilon\mu\omega^2 H_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} + \varepsilon_0\mu_0\omega_0^2 H_z = 0 \qquad \frac{\partial}{\partial t} = j\omega, \ \frac{\partial}{\partial z} = \gamma$$

$$\frac{\partial H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (\gamma^2 + \varepsilon_0\mu_0\omega_0^2) H_z = 0 \quad \text{on pose } \gamma^2 + \varepsilon_0\mu_0\omega_0^2 = k_c^2$$

$$\gamma^2 = -k_g^2, \quad \omega_0^2\varepsilon_0\mu_0 = k_0^2 \Rightarrow -k_g^2 + k_0^2 = k_c^2 \Rightarrow k_c^2 = k_0^2 - k_g^2$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k_c^2 H_z = 0 \Rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (k_0^2 - k_g^2) H_z = 0$$

$$k_c^2 = \gamma^2 + \varepsilon_0\mu_0\omega_0^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

- f_c pour $TE_{10} \Rightarrow m = 1, n = 0$

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{c}{2\cancel{\pi}}\cdot\frac{\cancel{\pi}}{a} = \frac{3.10^8}{2\times 2.10^{-2}} = 0{,}75.10^{10}\ Hz = 7{,}5\ GHz$$

- f_c pour $TE_{01} \Rightarrow m = 0, n = 1$

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{b}\right)^2} = \frac{c\cdot\cancel{\pi}}{2\cancel{\pi}\cdot b} = \frac{3.10^8}{2.10^{-2}} = 1{,}5.10^{10}\ Hz = 15\ GHz$$

- f_c pour $TE_{20} \Rightarrow m = 2, n = 0$

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{2\pi}{a}\right)^2 + 0} = \frac{c}{\cancel{2\pi}}\left(\frac{\cancel{2\pi}}{\cancel{2}.10^{-2}}\right) = \frac{3.10^8}{2.10^{-2}} = 15\ GHz$$

Exercice n°: 1

- Equations de Maxwell.

$$\overrightarrow{\text{rot}\,E} = -\mu \frac{\partial H}{\partial t}$$

$$\overrightarrow{\text{rot}\,H} = \varepsilon_0 \frac{\partial E}{\partial t}$$

$\text{div}\, E = \frac{\rho}{\varepsilon_0}$, $\text{div}\, E = 0$ (loin des charges)

$\text{div}\, B = 0$, $\text{div}\, \mu H = 0 \Rightarrow \text{div}\, H = 0$

ona $\Delta E_z = -\mu \varepsilon \omega^2 E_z$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\mu_0 \varepsilon_0 \omega^2 E_z \qquad \frac{\partial E}{\partial z} = -\gamma E_z$$

$$\frac{\partial^2 E(x,y)}{\partial x^2} + \frac{\partial^2 E(x,y)}{\partial y^2} + (\gamma^2 + \omega^2 \varepsilon_0 \mu_0) E(x,y) = 0 \qquad \frac{\partial^2 E}{\partial z^2} = \gamma^2 E_z$$

La Solution est de la forme.

$E(x,y) = X.Y.$

$$\frac{d^2 X.Y}{dx^2} + \frac{d^2 X.Y}{dy^2} + (k_c^2) X.Y = 0$$

$$Y \frac{d^2 X}{dx^2} + X \frac{d^2 Y}{dy^2} + k^2 XY = 0$$

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} + k^2 = 0 \qquad \text{on pose } k^2 = A^2 + B^2$$

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} = -A^2 - B^2$$

$+\frac{1}{X}\frac{d^2X}{dx^2} = -A^2 \Rightarrow X = C_1 \cos Ax + C_2 \sin Ax.$

$\frac{1}{Y}\frac{d^2Y}{dy^2} = -B^2 \Rightarrow Y = C_3 \cos By + C_4 \sin By$

$E = X.Y$, pour $x = 0 \Rightarrow E = 0$

$(\underbrace{C_1 \cos Ax}_{1} + C_2 \cancel{\sin Ax}^{0})(C_3 \cos By + C_4 \sin By) = 0$

$\Rightarrow C_1 = 0$

$(C_1 \cos Ax + C_2 \sin Ax)(C_3 \underbrace{\cos By}_{1} + 0) = 0 \Rightarrow C_3 = 0$

$E = E_0 \sin Ax \sin By.$ pour $x = a \Rightarrow E = 0$

$\sin A.a = 0 \Rightarrow A.a = m\pi \Rightarrow A = \frac{m\pi}{a}.$

$y = b \Rightarrow E = 0$

$\sin B.y = 0 \Rightarrow By = n\pi \Rightarrow B = \frac{n\pi}{b}.$

donc

$E = E_0 \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y.$

$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \Rightarrow f_c = \frac{c}{2\pi}\sqrt{\left(\frac{2\pi}{7.10^{-2}}\right)^2 + \left(\frac{\pi}{3.10^{-2}}\right)^2}$