TNS

Rania Dj

- $\boxed{f_e \geq f_{max}}$.

Signal échantillonné :

$$\boxed{x_e(t) = x(t).\delta_{T_e}(t)}$$

$$\boxed{\delta_{T_e}(t) = \delta(t-T_e)}$$

$$\boxed{TF[\delta_{T_e}(t)] = F_e \sum_{-\infty}^{+\infty} \delta(f - kF_e)}$$

$$\boxed{X_e(f) = F_e \sum_{-\infty}^{+\infty} X(f - kF_e)}$$

- Fréquence de coupure :

$$\boxed{f_c = \frac{F_e}{2}}$$

- Fréquence de Nyquist :

$$\boxed{f_{Nyq} = 2f_0} \qquad \boxed{f_{Nyq} = 2f_{max}}$$

- Pour récupérer le signal en bande étroite on fait filtre passe bande

Série de fourier d'un signal discret :

$$S(n) = \sum_{k=\langle N\rangle} S_k e^{j2\pi \frac{k}{N} n}$$

$$S_k = \frac{1}{N} \sum_{k=\langle N\rangle} S(n) e^{-j2\pi \frac{k}{N} n}$$

Exemple ①

$$S(n) = \cos\left(2\pi \frac{1}{7} n\right)$$

calculer S_k :

$\cancel{S(n) = \frac{1}{2}\cos}$ $\qquad N = 7$

$$S(n) = \frac{1}{2} e^{-j2\pi \frac{1}{7} n} + \frac{1}{2} e^{j2\pi \frac{1}{7} n}$$

alors : $\boxed{S_1 = S_{-1} = \frac{1}{2}}$

Exemple ② :

$$S(n) = \sin^2 2\pi \frac{1}{20} n.$$

$$\boxed{\sin^2(a) = \frac{1 - \cos(2a)}{2}}$$

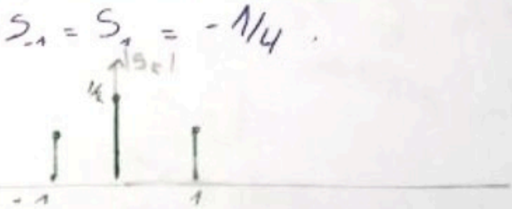
$$S(n) = \frac{1}{2} - \frac{\cos\left(2\pi \frac{1}{10}\right) n}{2}$$

$$S(n) = \frac{1}{2}\left[1 - \cos 2\pi \frac{1}{10} n\right]$$

$$S(n) = \frac{1}{2}\left[1 - \left(\frac{1}{2} e^{-j2\pi \frac{1}{10} n} + \frac{1}{2} e^{j2\pi \frac{1}{10} n}\right)\right]$$

$$\boxed{S(n) = \frac{1}{2} - \frac{1}{4} e^{-j2\pi \frac{1}{10} n} - \frac{1}{4} e^{j2\pi \frac{1}{10} n}}$$

$S_0 = 1/2$, $S_{-1} = S_1 = -1/4$.



Exemple ③ :

$$S(n) = \cos^3 \pi \frac{1}{5} n.$$

$$S(n) = \cos^3 2\pi \frac{1}{10} n$$

$$S(n) = \left[\cos\left(2\pi \frac{1}{10} n\right)\right].\left[\cos^2\left(2\pi \frac{1}{10} n\right)\right]$$

$$S(n) = \frac{1}{2}\left[e^{-j2\pi \frac{1}{10} n} - e^{j2\pi \frac{1}{10} n}\right].\left[\frac{1 + \cos 2\pi \frac{1}{5} n}{2}\right]$$

$$S(n) = \frac{1}{2}\left[e^{-j2\pi \frac{1}{10} n} - e^{j2\pi \frac{1}{10} n}\right].\frac{1}{2}\left[1 + \frac{1}{2} e^{-j2\pi \frac{1}{5} n} + \frac{1}{2} e^{j2\pi \frac{1}{5} n}\right]$$

$$S(n) = \frac{1}{2}\left[e^{-j2\pi \frac{1}{10} n} - e^{j2\pi \frac{1}{10} n}\right].\left[\frac{1}{2} + \frac{1}{4} e^{-j2\pi \frac{1}{5} n} + \frac{1}{4} e^{j2\pi \frac{1}{5} n}\right]$$

Rama Dj

$$s(n) = \frac{1}{4}e^{-J2\pi\frac{1}{10}n} + \frac{1}{8}e^{-J2\pi\frac{3}{10}n} - \frac{1}{4}e^{J2\pi\frac{1}{10}n} - \frac{1}{8}e^{J2\pi\frac{3}{10}n}$$

$s(n)$: $S_{-1} = S_1 = \frac{3}{8}$

$S_3 = S_{-3} = 1/8$

Exo 3 : série ② :

Transformée de fourier :

①

$$s(n) = \begin{cases} 1 & -11 < n < 11 \\ 0 & \text{ailleurs} \end{cases}$$

$$\boxed{s(f) = \sum_{-\infty}^{+\infty} s(n)\, e^{-J2\pi f n}}$$

$$s(f) = \sum_{-10}^{10} 1\, e^{-J2\pi f n}$$

- Si $f = 0, \pm 1, \pm 2 \ldots\ldots$

$$s(f) = \sum_{-10}^{10} (1)^n \Rightarrow s(f) = 10 - (-10) + 1$$

$$\boxed{s(f) = 21}$$

- Si $f \neq 0, \pm 1, \pm 2 \ldots$

$$s(f) = \sum_{-10}^{10} 1\, e^{-J2\pi f n}$$

$$s(f) = e^{+J2\pi f(10)} \cdot \frac{1 - e^{-J2\pi f(21)}}{1 - e^{-J2\pi f}}$$

$$s(f) = e^{J2\pi f 10} \cdot \frac{e^{-J\pi f 21}\left(e^{J\pi f 21} - e^{-J\pi f 21}\right)}{e^{-J\pi f}\left(e^{+J\pi f} - e^{J\pi f}\right)}$$

$$s(f) = \cancel{e^{J2\pi f(10)}} \cdot \cancel{e^{J\pi f}} \cdot \cancel{e^{-J\pi f 21}} \cdot \left[\frac{e^{J\pi f 21} - e^{-J\pi f 21}}{e^{J\pi f} - e^{-J\pi f}}\right]$$

$$s(f) = \frac{2J\sin(\pi f 21)}{2J\sin(\pi f 21)}$$

$$\boxed{s(f) = \frac{\sin(21\pi f)}{\sin(21\pi f)}}$$

②

$$s(n) = \begin{cases} n & \text{pour } -3 < n < 3 \\ 0 & \text{ailleurs} \end{cases}$$

$$s(f) = \sum_{-2}^{2} n\, e^{-J2\pi f n}$$

Si $f = 0, \pm 1, \pm 2 \ldots\ldots$

$$\cancel{s(f) = \sum_{-2}^{2} (n)^n}$$

$\cancel{s(f) = S_n}$ on pose : $2\alpha = -J2\pi f$

$$\boxed{\alpha = -J\pi f}$$

$$s(f) = \sum_{n=-2}^{2} n\, e^{\alpha n}, \quad n e^{\alpha n} = \frac{d}{d\alpha} \underbrace{e^{\alpha n}}_{I}$$

Si : $f = 0; \pm 1; \pm 2 \ldots\ldots$

Posons : $m = n + 2 \longrightarrow n = -2, m = 0$

$n = m - 2 \longrightarrow n = 2, m = 4$

$$I = \sum_{m=0}^{4} 1 = 5$$

et $s(f) = \frac{d}{d\alpha} 5 = 0.$

Si $f \neq 0; \pm 1; \pm 2; \pm 3 \ldots\ldots$

$$I = \sum_{m=0}^{4} (e^{\alpha})^{m-2}, \quad (e^{\alpha})^{-2} \cdot \sum_{m=0}^{4} (e^{\alpha})^m$$

$$I = (e^{\alpha})^{-2} \cdot \frac{1 - (e^{\alpha})^5}{1 - e^{\alpha}}$$

$$s(f) = \frac{dI}{d\alpha} = 0.$$

$$\text{Donc : } s(f) = \begin{cases} 0 & f = 0; \pm 1; \pm 2 \ldots \\ 0 & f \neq 0; \pm 1; \pm 2 \ldots \end{cases}$$

$$s(n) = \begin{cases} \cos 2\pi \frac{1}{10} n & -10 < n < 10 \\ 0 & \text{ailleurs} \end{cases}$$

Rania Dj

$$S(f) = \sum_{-9}^{9} s(n)\, e^{-j2\pi f n}.$$

$$S(f) = \sum_{-9}^{9} \cos(2\pi \tfrac{1}{10} n) \cdot e^{-j2\pi f n}.$$

$$S(f) = \sum_{-9}^{9} \left(\frac{e^{-j2\pi \frac{1}{10} n} + e^{j2\pi \frac{1}{10} n}}{2} \right) e^{-j2\pi f n}.$$

$$S(f) = \sum_{-9}^{9} \left[\frac{e^{-j2\pi (f + \frac{1}{10}) n} + e^{-j2\pi (f - \frac{1}{10}) n}}{2} \right]$$

$$S(f) = \sum_{-9}^{9} \frac{1}{2} e^{-j2\pi (f + \frac{1}{10}) n} + \sum_{-9}^{9} \frac{1}{2} e^{-j2\pi (f - 1/10) n}.$$

$$S(f) = \underbrace{\frac{1}{2} \sum_{-9}^{9} e^{-j2\pi (f + \frac{1}{10}) n}}_{I_1} + \underbrace{\frac{1}{2} \sum_{-9}^{9} e^{-j2\pi (f - 1/10) n}}_{I_2}.$$

$$I = \frac{1}{2} \sum_{-9}^{9} e^{-j2\pi (f + 1/10) n}$$

on pose : $P = f + 1/10$

Si $P = 0, \pm 1, \pm 2 \dots$

$$I_1 = \frac{1}{2} \sum_{-9}^{9} (1)^n$$

$$\boxed{I_1 = \frac{19}{2}}$$

Si $P \neq 0, \pm 1, \pm 2 \dots$

$$I_1 = \frac{1}{2} \sum_{-9}^{9} e^{-j2\pi (f + 1/10) n}.$$

$$I_1 = \frac{1}{2} \left[e^{-j2\pi (f + \frac{1}{10})(-9)} \cdot \frac{1 - e^{-j2\pi (f + 1/10)(19)}}{1 - e^{-j2\pi (f + 1/10)}} \right]$$

$$\boxed{I_1 = \frac{1}{2} \frac{\sin 2\pi (f + 1/10)\, 19}{\sin 2\pi (f + 1/10)}}$$

La même chose avec I_2 :

$$\boxed{I_2 = \frac{1}{2} \frac{\sin 2\pi (f - 1/10)\, 19}{\sin 2\pi (f - 1/10)}}$$

$$\boxed{I_2 = \frac{19}{2}} \quad \text{si } P = 0, \pm 1 \dots$$

$$\cancel{S(f) = \begin{cases} \frac{19}{2} + \frac{19}{2} & \text{si } P = 0, \pm 1, \pm 2 \dots \\ \frac{1}{2} \left[\frac{\sin \pi (f + 1/10)\, 19}{\sin \pi (f + 1/10)} + \frac{\sin \pi (f - 1/10)\, 19}{\sin \pi (f - 1/10)} \right] & \text{si } P \neq 0, \pm 1. \end{cases}}$$

$$S(f) = \begin{cases} \frac{1}{2} \frac{\sin \pi (f + 1/10)\, 19}{\sin \pi (f + 1/10)} & \dots \text{si } f \neq P - \frac{1}{10} \\ \frac{1}{2} \frac{\sin \pi (f - 1/10)\, 19}{\sin \pi (f - 1/10)} & \dots \text{si } f = P + 1/10 \\ \frac{19}{2} & \text{si } f = P \pm \frac{1}{10} \end{cases}$$

Exo 4 série 2

$$e(n) = \exp(j2\pi \tfrac{k}{N} n)$$

$$E(f) = \sum_{-\infty}^{+\infty} \left(e^{j2\pi \frac{k}{N} n} \right) \left(e^{-j2\pi f n} \right)$$

en analogique :

$$e(t) = e^{j2\pi f_0 t}, \quad TF[e(t)]$$

$$\boxed{E(f) = \delta(f - f_0)}$$

en numérique :

$TF[e(n)] = \delta(f - \frac{k}{N})$ ⇒ sur une seul periode.

$$\boxed{TF[e(n)] = \sum_{-\infty}^{+\infty} \delta(f - \tfrac{k}{N} - P)}$$

2 : $s(n) = \sum_{k \in [N]} S_k\, e^{j2\pi \frac{k}{N} n}.$

$$S(f) = \sum_{-\infty}^{+\infty} s(n)\, e^{-j2\pi \frac{k}{N} n}.$$

Donc : $\cancel{S(f) = \sum_{k=[N]} \sum_{l=-\infty}^{+\infty} S_k e^{j2\pi \frac{k}{N} n} \cdot e^{-j2\pi \frac{k}{N} n}}$

$$S(f) = TF[s(n)].$$

$$S(f) = \sum_{k=[N]} \sum_{-\infty}^{+\infty} S_k\, \delta(f - \tfrac{k}{N} - P)$$

$$S(f) = \sum_{k=[N]} S_k \sum_{-\infty}^{+\infty} \delta(f - \tfrac{k}{N} - P)$$ ⇒ TF d'un signal périodique

3\

$$S_N(n) = \begin{cases} 1 & -N_1 + 1 \leq n \leq N_1 + 1 \\ 0 & \text{ailleurs.} \end{cases} \Rightarrow$$

$$S(f) = \sum_{K=[N]} S_K \sum_{-\infty}^{+\infty} \delta\left(f - \frac{K}{N}\right) \Rightarrow \text{une seule période}$$

$$S_K = \frac{1}{N} \sum_{K=[N]} s(n) e^{-j2\pi \frac{K}{N} n}$$

$$S_K = \frac{1}{N} \sum_{-N_1+1}^{N_1-1} 1\, e^{-j2\pi \frac{K}{N} n} \; ; \; \boxed{S_K = \frac{1}{N} \sum_{K[N]} S_1(n) e^{-j2\pi \frac{K}{N} n}}$$

$$S_K = \frac{1}{N} \sum_{-N_1+1}^{N_1-1} e^{-j2\pi \frac{K}{N} n}$$

Si $K = 0; \pm N; \pm 2N; \pm 3N \ldots$

$$S_K = \frac{1}{N} \sum_{-N_1+1}^{N_1-1} (1)^n \Rightarrow \boxed{S_K = \frac{2N_1 - 1}{N}}$$

Si $K \neq 0; \pm N; \pm 2N; \pm 3N \ldots$

$$S_K = \frac{1}{N} \sum_{-N_1+1}^{N_1-1} \left(e^{-j2\pi \frac{K}{N}}\right)^n$$

$$S_K = \frac{1}{N} \left[\frac{\sin\left(\pi \frac{K}{N}\right) 2N_1 - 1}{\sin \pi K/N} \right]$$

Donc :

$$S(f) = \begin{cases} \sum_{K=[N]} \left(\frac{2N_1 - 1}{N}\right) \sum_{-\infty}^{+\infty} \delta\left(f - \frac{K}{N}\right) \ldots \text{ si } K = 0, \pm N \ldots \\ \sum_{K[N]} \frac{\sin\left(\pi \frac{K}{N}\right) 2N_1 - 1}{N \sin \pi \frac{K}{N}} \sum_{-\infty}^{+\infty} \delta\left(f - \frac{K}{N}\right) \ldots \text{ si } K \neq 0, \pm N \ldots \end{cases}$$

4)

$$s(n) = \cos\left(2\pi \frac{1}{10} n\right)$$

$$S_K = \frac{1}{N} \sum_{K=[N]} s(n) e^{-j2\pi \frac{K}{N} n}$$

$$s(n) = \sum_{K=[N]} S_K e^{j2\pi \frac{K}{N} n},$$

$\boxed{N = 10}$

$S_K = 1/2$; $S_1 = S_{-1} = 1/2$

$$S_K = \begin{cases} \frac{1}{2} & K = 1 \\ \frac{1}{2} & K = -1 \\ 0 & \text{ailleurs} \end{cases}$$

$$\boxed{S(f) = \frac{1}{2} \sum \delta\left(f - \frac{1}{10} - \ell\right) + \frac{1}{2} \sum \delta\left(f + \frac{1}{10} - \ell\right)}$$

② $s(n) = \sin^2\left(2\pi \frac{1}{20} n\right)$

$S_0 = \frac{1}{2}$, $S_1 = S_{-1} = -\frac{1}{4}$

$$S_K = \begin{cases} \frac{1}{2} & K = 0 \\ -\frac{1}{4} & K = 1 \\ -\frac{1}{4} & K = -1 \end{cases}$$

$$S(f) = \frac{1}{2} \sum_{-\infty}^{+\infty} \delta(f - \ell) - \frac{1}{4} \sum_{-\infty}^{+\infty} \delta\left(f - \frac{1}{20} - \ell\right) - \frac{1}{4} \sum_{-\infty}^{+\infty} \delta\left(f + \frac{1}{20} - \ell\right)$$

③ $s(n) = \cos^3 \pi \, 1/5 \, n$ $\boxed{N = 10}$

$$S_K : \left.\begin{aligned} S_{-1} = S_1 &= 3/8 \\ S_{-3} = S_3 &= 1/8 \end{aligned}\right\}$$

$$S(f) = \frac{3}{8} \sum_{-\infty}^{+\infty} \delta\left(f - \frac{1}{10} - \ell\right) + \frac{3}{8} \sum_{-\infty}^{+\infty} \left(f + \frac{1}{10} - \ell\right) + \frac{1}{8} \sum_{-\infty}^{+\infty} \delta\left(f - \frac{3}{10} - \ell\right) + \frac{1}{8} \sum_{-\infty}^{+\infty} \left(f + \frac{3}{10} - \ell\right).$$

La DFT

Exo 01 série ③

1. $s(n) = 1$ $\quad n = 0, 1, 2 \ldots N-1$.

$$\boxed{S(K) = \sum_{n=0}^{N-1} s(n) e^{-j2\pi \frac{K}{N} n}}$$

$$S(K) = \sum_{n=0}^{N-1} e^{-j2\pi \frac{K}{N} n}.$$

- Si $K = 0, \pm N, \pm 2N \ldots$

$$S(K) = \sum_{n=0}^{N-1} (1)^n = N$$

- Si $K \neq 0, \pm N, \pm 2N \ldots$

$$S(K) = \left(e^{-j2\pi \frac{K}{N}}\right)^0 \cdot \frac{1 - \left(e^{-j2\pi \frac{K}{N}}\right)^N}{1 - e^{-j2\pi \frac{K}{N}}} = 0.$$

$S(K) = 0$

$$S(K) = \begin{cases} N & \text{si } K = 0; \pm N; \pm 2N \ldots \\ 0 & \text{si } K \neq 0, \pm N, \pm 2N \ldots \text{ ailleurs.} \end{cases}$$

Romaissa Dj

④

Rama Dj

DfT :

$s(n) = 1 \quad , n = 0, 1, 2 \ldots ; 20$

$N - 1 = 20 , \boxed{N = 21}$

$S(k) = \sum_{n=0}^{20} \left(e^{-J 2\pi \frac{k}{N}}\right)^n$

Si $K = 0 ; \pm 21 ; \pm 42 \ldots$

$\boxed{S(K) = 21}$

Si $K \neq 0 ; \pm 21 ; \pm 42 \ldots$

$S(K) = 0.$

2) $s(n) = \cos 2\pi \frac{1}{10} n \quad ; n = 0, 1, 2, \ldots N-1$

$S(k) = \sum_{0}^{N-1} \cos\left(2\pi \frac{1}{10} n\right) e^{-J 2\pi \frac{k}{N} n}$

$S(k) = \sum_{0}^{N-1} \left(\frac{1}{2} e^{-J 2\pi \frac{1}{10} n} + \frac{1}{2} e^{J 2\pi \frac{1}{10} n}\right) e^{-J 2\pi \frac{k}{N} n}$

$S(k) = \sum_{0}^{N-1} \left(\underbrace{\frac{1}{2} e^{-J 2\pi \left(\frac{k}{N} + \frac{1}{10}\right) n}}_{I_1} + \underbrace{\frac{1}{2} e^{-J 2\pi \left(\frac{k}{N} - \frac{1}{10}\right) n}}_{I_2}\right)$

$I_1 = \frac{1}{2} \sum_{0}^{N-1} e^{-J 2\pi \left(\frac{k}{N} + \frac{1}{10}\right) n}$

Si $K = -\frac{N}{10}$

$S(k) = \boxed{I_1 = \frac{N}{2}}$

Si $K \neq -\frac{N}{10}$

$$\boxed{I_1 = \frac{1}{2} \frac{\sin J\pi \left(\frac{k}{N} + \frac{1}{10}\right) N}{\sin J\pi \left(\frac{k}{N} + \frac{1}{10}\right)}}$$

$I_2 = \frac{1}{2} \sum_{n=0}^{N-1} e^{-J 2\pi \left(\frac{k}{N} - \frac{1}{10}\right) n}$

Si $\Rightarrow K = \frac{N}{10}$

$\boxed{I_2 = \frac{N}{2}}$

Si $K \neq \frac{N}{10}$

$$\boxed{I_2 = \frac{1}{2} \frac{\sin J\pi \left(\frac{k}{N} - \frac{1}{10}\right) N}{\sin J\pi \left(\frac{k}{N} - \frac{1}{10}\right)}}$$

Donc :

$$S(k) = \begin{cases} \frac{N}{2} & \text{si} \ldots \quad K = \pm \frac{N}{10} \\ \frac{1}{2} \frac{\sin J\pi \left(\frac{k}{N} + \frac{1}{10}\right) N}{\sin J\pi \left(\frac{k}{N} + \frac{1}{10}\right)} \ldots & \text{si } K \neq -\frac{N}{10} \\ \frac{1}{2} \frac{\sin J\pi \left(\frac{k}{N} - \frac{1}{10}\right) N}{\sin J\pi \left(\frac{k}{N} - \frac{1}{10}\right)} e^{J\pi \left(\frac{k}{N} - \frac{1}{10}\right) - N - 1} \ldots & \text{si } K \neq +\frac{N}{10} \end{cases}$$

3) $s(n) = a^n \quad , n = 0, 1, 2 \ldots, 20 , \ |a| < 1$

$S(k) = \sum_{n=0}^{20} a^n e^{-J 2\pi \frac{k}{N} n}$

$S(k) = \sum_{n=0}^{20} \left(a e^{-J 2\pi \frac{k}{N}}\right)^n$

$S(k)$, si $K = 0 ; \pm N ; \pm 2N \ldots$

$S(k) = \sum_{n=0}^{20} (a)^n$

$S(k) = a^0 \frac{1 - a^{21}}{1 - a}$

$$\boxed{S(k) = \frac{1 - a^{21}}{1 - a}}$$

Si $k \neq 0 ; \pm N ; \pm 2N$

$$\boxed{S(k) = \frac{1 - \left(a e^{-J 2\pi \frac{k}{N}}\right)^{21}}{1 - a e^{-J 2\pi \frac{k}{N}}}}$$

Exo 2 série 3

Exo ② série ③ Rania Dj

$s_1(n) = \cos^2 4\pi \frac{1}{N} n$; $n = 0, 1, 2 \dots N-1$

$S(k) = \sum_{n=0}^{N-1} s_1(n) e^{-j2\pi \frac{k}{N} n}$

$S(k) = \sum_{n=0}^{N-1} \cos^2(4\pi \frac{1}{N} n) e^{-j2\pi \frac{k}{N} n}$

$S(k) = \sum_{n=0}^{N-1} \left[\left(\frac{1 + \cos 4\pi \frac{2}{N} n}{2} \right) e^{-j2\pi \frac{k}{N} n} \right]$

$S(k) = \sum_{n=0}^{N-1} \left[\left(\frac{1}{2} + \left(\frac{1}{4} e^{-j2\pi \frac{4}{N} n} + \frac{1}{4} e^{j2\pi \frac{4}{N} n} \right) \right) e^{-j2\pi \frac{k}{N} n} \right]$

$\cancel{S(n) = \sum_{n=0}^{N-1} \frac{1}{2} e^{-j2\pi \frac{k}{N} n} - \frac{1}{4} e}$

$\boxed{s(n) = \frac{1}{2} + \frac{1}{4} e^{j2\pi \frac{4}{N} n} + \frac{1}{4} e^{-j2\pi \frac{4}{N} n}}$

$S_k = \begin{cases} \frac{1}{2} & k = 0 \\ \frac{1}{4} & k = 4 \\ \frac{1}{4} & k = -4. \end{cases}$

$\boxed{S(k) = N S_k}$

$S(k) = \begin{cases} \frac{N}{2} & \text{si } k = 0 \\ \frac{N}{4} & \text{si } k = 4 \\ \frac{N}{4} & \text{si } k = N-4 \end{cases}$

2). $s_2(n) = \sin^2 4\pi \frac{1}{N} n$; $n = 0, 1, 2 \dots N-1$;

$s_2(n) = \frac{1 - \cos(2\pi \frac{4}{N} n)}{2}$

$s_2(n) = \frac{1}{2} - \frac{1}{4} e^{j2\pi \frac{4}{N} n} - \frac{1}{4} e^{-j2\pi \frac{4}{N}}$.

$S_k = \begin{cases} \frac{1}{2} & \text{si } k = 0 \\ -\frac{1}{4} & \text{si } k = 4 \\ -1/4 & \text{si } k = -4. \end{cases}$

$S(k) = N S_k$.

$S(k) = \begin{cases} \frac{N}{2} & \text{si } k = 0 \\ -\frac{N}{4} & \text{si } k = 4 \\ \frac{N}{4} & \text{si } k = -4 \end{cases}$

3). $s_3(n) = \cos^2 \pi \frac{1}{10} n$; $n = 0, 1, 2, \dots$

$s_3(n) = \frac{1 + \cos 2\pi \frac{1}{10} n}{2}$

$s_3(n) = \frac{1}{2} + \frac{1}{4} e^{-j2\pi \frac{1}{10} n} + \frac{1}{4} e^{j2\pi \frac{1}{10} n}$.

$S_k = \begin{cases} \frac{1}{2} & \text{si } k = 0 \\ \frac{1}{4} & \text{si } k = -1 \\ \frac{1}{4} & \text{si } k = 1. \end{cases}$

$N = 10$

$S(k) = N S_k$.

$S(k) = \begin{cases} \frac{N}{2} & k = 0 \\ \frac{N}{4} & k = +1 \\ \frac{N}{4} & k = -1 + 2N \end{cases}$

$S(k) = \begin{cases} 5 & k = 0 \\ \frac{10}{4} & k = 1 \\ \frac{10}{4} & k = 19. \end{cases}$

Exo 3

$S(k) = 21$, $k = 0, 21, 42, \dots$

DFT inverse.

$s(n) = \frac{1}{N} \sum_{k=0}^{N-1} S(k) e^{+j2\pi \frac{k}{N} n}$

①. $s(n) = \frac{1}{21} \sum_{k=0}^{20} 21 e^{-j2\pi \frac{k}{N} n}$

$\boxed{s(n) = 1}$

②. $S(k) = 10$, $k = 1$ $k = 19$.

$s(n) = \frac{1}{N} \sum_{k=0}^{N-1} S(k) e^{+j2\pi \frac{k}{N} n}$

$s(n) = \frac{1}{N} \left[\sum_{k=0}^{N-1} 10 e^{+j2\pi \frac{1}{N} n} + \sum_{k=0}^{N-1} 10 e^{+j2\pi \frac{19}{N} n} \right]$

$s(n) = \frac{10}{N} \left(e^{-j2\pi \frac{1}{N}} + e^{+j2\pi \frac{19}{N} n} \right)$

$N = 20$

$\boxed{s(n) = \frac{1}{2} \left(e^{j2\pi \frac{1}{20} n} + e^{-j2\pi \frac{1}{20} n} \right)}$

$\boxed{s(n) = \cos(2\pi \frac{1}{20} n)}$

$s(k) = N \qquad k = 0$

Rama Dj

$s(n) = \frac{1}{N} \sum_{k=0}^{N-1} s(k) e^{j2\pi \frac{k}{N} n}$

$s(n) = \frac{1}{N} \sum_{k=0}^{N-1} N e^{j2\pi \frac{k}{N}}$

$s(n) = 1$ avec $n = 0, 1, 2 \ldots N-1$

4| $s(k) = 1 \quad , \quad k = 0, \ldots N-1$

$s(n) = \frac{1}{N} \sum_{k=0}^{N-1} s(k) e^{j2\pi \frac{k}{N} n}$

$s(n) = \frac{1}{N} \sum_{k=0}^{N-1} e^{j2\pi \frac{k}{N} n}$

Pour $k = 0$

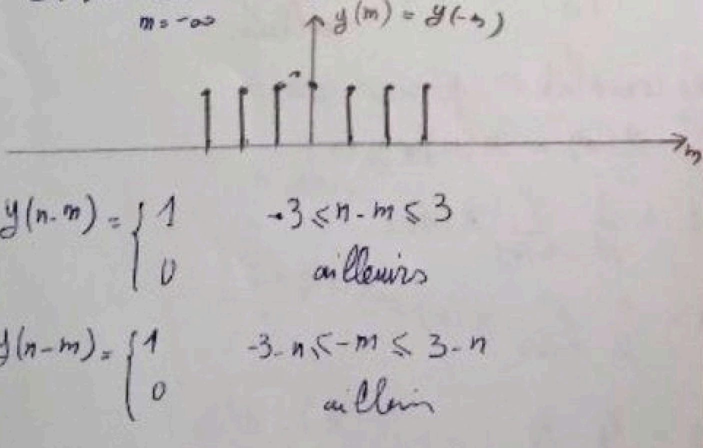
$s(n) = \frac{1}{N} N \Rightarrow \boxed{s(n) = 1}$

Exo① série ④

$x(n) = \begin{cases} 1 & -5 \leq n \leq 5 \\ 0 & \text{ailleurs} \end{cases} \quad \Big| \quad y(n) = \begin{cases} 1 & -3 \leq n \leq 3 \\ 0 & \text{ailleurs} \end{cases}$

$Z(n) = x(n) * y(n)$

$Z(n) = \sum_{m=-\infty}^{+\infty} x(m)\, y(n-m)$



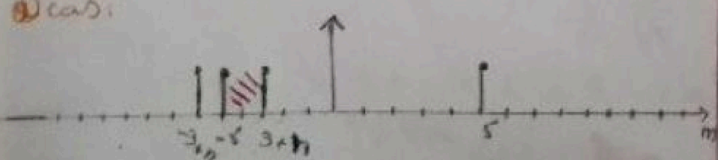
$y(n-m) = \begin{cases} 1 & -3 \leq n-m \leq 3 \\ 0 & \text{ailleurs} \end{cases}$

$y(n-m) = \begin{cases} 1 & -3-n \leq -m \leq 3-n \\ 0 & \text{ailleurs} \end{cases}$

$y(n-m) = \begin{cases} 1 & -3+n \leq m \leq 3+n \\ 0 & \text{ailleurs} \end{cases}$

① cas 50

② cas:



$-3+n < -5$

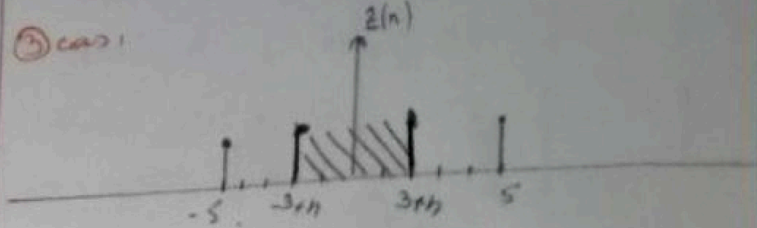
$n < -2$

$-5 \leq 3+n \; , \; n > -8 \qquad -8 \leq n < -2$

$Z(n) = \sum_{-5}^{3+n} 1 \times 1 \Rightarrow Z(n) = 3+n+5+1$

$\boxed{Z(n) = n+9}$

③ cas:



$-3+n > -5 \; ; \; n > -2$

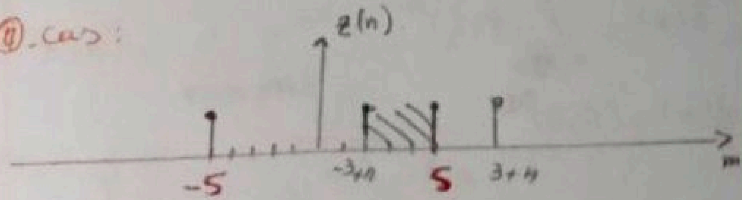
$3+n < 5 \; ; \; n < 2$

$-2 \leq n \leq 2$

$Z(n) = \sum_{-3+n}^{3+n} 1 \; , \; Z(n) = 3+n+3-n+1$

$\boxed{Z(n) = 7}$

④ cas:



$Z(n) = \sum_{-3+n}^{5} 1$

$Z(n) = 5+3-n+1 \Rightarrow \boxed{Z(n) = 9-n}$

$-3+n \leq 5 \qquad n \leq 8$

$3+n \geq 5 \; , \quad n \geq 2$

$2 < n \leq 8$

⑤ cas: $\quad n > 8, \; Z(n) = 0$

2|:

$x(n) = \begin{cases} 1 & 0 < n < 20 \\ 0 & \text{ailleurs} \end{cases} , \quad y(n) = \begin{cases} 0{,}6^{|n|} & -10 \leq n \leq 10 \\ 0 & \text{ailleurs} \end{cases}$

$Z(n) = x(n) * y(n)$

$Z(n) = \sum_{m=-\infty}^{+\infty} y(m) . x(n-m)$

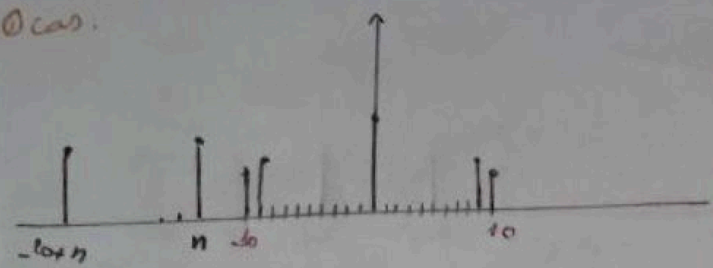
$x(m)\, x(n-m) = \begin{cases} 1 & 0 < n-m < 20 \\ 0 & \text{ai} \end{cases}$

$x(n-m) = \begin{cases} 1 & -n < -m < 20-n \\ 0 & \text{ailleurs} \end{cases}$

$$x(n-m) = \begin{cases} 1 & -20+n < m < n \\ 0 & \text{ailleurs} \end{cases}$$

Rania Dj

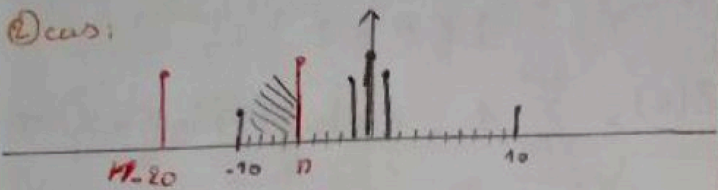
① cas.



$n < -10$

$Z(n) = 0$

② cas:



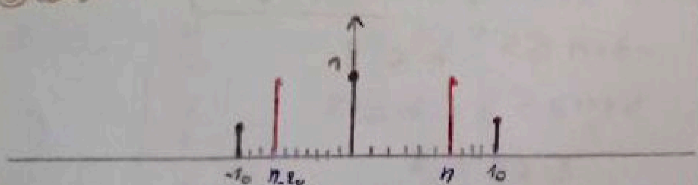
$-10 \leq n \leq 10$

$Z(n) = \sum_{m=-10}^{n} (0,6)^n$

$Z(n) = (0,6)^{-10} \cdot \dfrac{1-(0,6)^{n+11}}{1-0,6}$

$\cancel{Z(n) = (0,6)^{n+11}}$

③ cas:



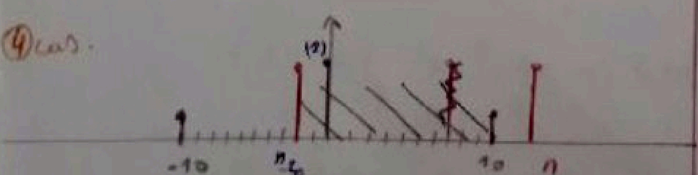
$n < 10 \quad , \quad -10 \leq n-20$

$-10 \leq n \qquad -10 \leq n < 10$

$Z(n) = \sum_{n-20}^{n} (0,6)^n$, $\cancel{Z(n) = (0,6)^{n-20}}$

$Z(n) = (0,6)^{n-20} \cdot \dfrac{1-(0,6)^{21}}{1-0,6}$

④ cas.



$\cancel{n > 30} \qquad \cancel{n-20 \leq 10}$

$n > 10 \qquad n \leq 30$

$\cancel{20 \leq n < 10}$

$\cancel{n-20 \geq 10}$, $n-20 < 0$, $n < 20$.

$Z(n) = \sum_{n-20}^{10} (0,6)^n$

$Z(n) = (0,6)^{n-20} \cdot \dfrac{1-(0,6)^{31-n}}{1-0,6} \qquad 10 \leq n < 30$

$n \geq 10$

$n-20 \leq 10$

$n < 30 \qquad n-20$

⑤ cas.

$Z(n) = 0 \qquad n-20 \not< 10 \;.\; n \not< 30.$

Exo ②

$Z(n) = x(n) * y(n)$, $\boxed{N = 21}$

$Z(n) = \sum_{m=-\infty}^{+\infty} x(n-m)\, y(m)$

$Z(n) = \sum_{m=-\infty}^{+\infty} y(m)\, x(n-m)$

$\cancel{x_N(n) = \begin{cases} 1 & -5 \leq n-m \leq 5 \\ 0 & \text{ailleurs} \end{cases}}$

$\cancel{x_N(n) = \begin{cases} 1 & -5-n \leq -m \leq 5-n \\ 0 & \text{ailleurs} \end{cases}}$

$\cancel{x_N(n-m) = \begin{cases} 1 & -5+n \leq m \leq 5+n \\ 0 & \text{ailleurs} \end{cases}}$

La convolution périodique:

$Z(n) = x(n) * y(n)$

$$\begin{cases} X_k = \dfrac{1}{N} \sum_{n=[N]} x(n)\, e^{-j2\pi \frac{k}{N} n} \\ Y_k = \dfrac{1}{N} \sum_{n=[N]} y(n)\, e^{-j2\pi \frac{k}{N} n} \end{cases}$$

$Z_k = X_k \cdot Y_k \Rightarrow \boxed{Z(k) = N Z_k}$

$z(n) = DFT^{-1}(Z(k))$

$X_k = \dfrac{1}{N} \sum_{-5}^{5} 1\, e^{-j2\pi \frac{k}{N} n} \Rightarrow$

Pour $k = 0, \pm N, \pm 2N$ ...

$X_k = \dfrac{1}{N} \sum_{-5}^{5} (1)\, e^{-j2\pi \frac{k}{N} n}$

$\boxed{X_k = \dfrac{11}{N}}$

$\frac{1}{N}\sum_{-5}^{5} n\, e^{-j2\pi\frac{k}{N}n}$ Ramia DJ

$k \neq 0, \pm N, \pm 2N \ldots$

$= \frac{1}{N} \frac{\sin(j\pi\frac{k}{N})\,11}{\sin(j\pi\frac{k}{N})}$

$) = \frac{1}{N}\sum_{-5}^{5} n e^{-j2\pi\frac{k}{N}n}$, $\alpha = -2j\frac{k}{N}$

$\boxed{n e^{\alpha n} = \frac{d e^{\alpha n}}{d\alpha}}$

Si $k = 0, \pm N, \pm 2N \ldots$

$y(n) = \frac{d}{d\alpha}\left[\frac{1}{N}\sum_{-5}^{5} e^{\alpha}\right] = \frac{d}{d\alpha}\left[\frac{1}{N}\sum_{-5}^{5} 1^n\right]$

$y(n) = \frac{d}{d\alpha}\frac{11}{N} = 0$.

Si $k \neq 0, \pm N, \pm 2N \ldots$

$y(n) = \frac{d}{d\alpha}\left[\frac{1}{N}\left(e^{-\alpha 5}\cdot\frac{1-e^{\alpha 11}}{1-e^{\alpha}}\right)\right]$

$y(n) = \frac{d}{d\alpha}\left[\frac{1}{N}\frac{\sin \pi\frac{k}{N}(11)}{\sin \pi k/N}\right]$

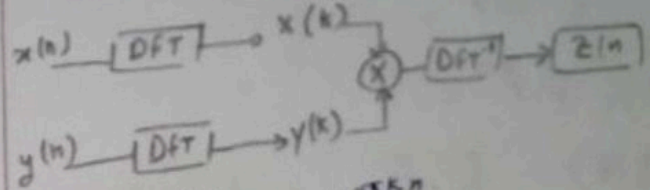
La convolution circulaire

Exo 3

1. $x(n) = 1$, $n = 0, 1, 2 \ldots N-1$

$y(n) = 1$, $n = 0, 1, 2 \ldots N-1$

$z(n) = x(n) * y(n)$



$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi\frac{k}{N}n}$

$X(k) = \sum_{n=0}^{N-1} e^{-j2\pi\frac{k}{N}n}$

Pour : $k = 0$; $\boxed{X(k) = N}$

" $k \neq 0$, $X(k) = \sum_{n=0}^{N-1} e^{-j2\pi\frac{k}{N}n}$

$X(k) = \frac{1 - e^{-j2\pi\frac{k}{N}N}}{1 - e^{-j2\pi\frac{k}{N}}} = 0$.

$X(k) = \begin{cases} N & \Rightarrow k = 0 \\ 0 & \text{ailleurs} \end{cases} = Y(k)$

$Z(k) = X(k) \cdot Y(k)$

$Z(k) = \begin{cases} N^2 & \text{si } k = 0 \\ 0 & \text{ailleurs} \end{cases}$

$* \; z(n) = \frac{1}{N}\sum_{k=0}^{N-1} N^2 e^{j2\pi\frac{k}{N}n} = \frac{1}{N}N^2$

$z(n) = N$ pour $n = 0, 1, 2 \ldots$

La théorème de la valeur initiale :

$$x(0) = \lim_{z \to +\infty} X(z)$$

$$X(z) = \frac{z}{z - 0{,}8}$$

$$x(0) = 0{,}8^0 = 1$$

$$\lim_{z \to +\infty} X(z) = \lim_{z \to +\infty} \frac{z}{z - 0{,}8} = 1$$

$$\lim_{z \to +\infty} X(z) = \lim_{z \to +\infty} \frac{\cancel{z}}{\cancel{z}(1 - \underbrace{\frac{0{,}8}{z}}_{0})} = 1$$

verifiver.

$a - b = c - d + e$

$$h(n) = \begin{cases} \frac{c}{a} & \text{———} \quad n = 0 \\ \frac{bc}{a^2} - \frac{d}{a} & \text{———} \quad n = 1 \\ \frac{b^{n-2}}{a^{n-2}}\left[\left[\frac{b^2 c}{a^3} - \frac{bd}{a^2}\right] + \frac{e}{c}\right] & - \quad n \geq 2 \end{cases}$$

$a + b = c + d + e$

$$h(n) = \begin{cases} \frac{c}{a} & \text{———} \quad n = 0 \\ \frac{d}{a} - \frac{bc}{a^2} & \text{———} \quad n = 1 \\ (-1)^n \frac{b^{n-2}}{a^{n-2}}\left[\left[e - \frac{b}{a}\left(d - \frac{bc}{a}\right)\right]\right] & - \quad n \geq 2 \end{cases}$$

stable : $\sum h(n) < +\infty$

$b < a$

$\frac{F_i}{F_i} \to$ FIR : $e - \frac{b}{a}\left(d - \frac{bc}{a}\right) = 0.$