University of M'sila

Faculty of: Technology **Common Base**

First Series of exercises

Exercise 01:

Given the vectors $\vec{V}_1$ *and* $\vec{V}_2$ *in an orthonormal basis* $(\vec{\imath},\vec{\jmath},\vec{k})$ *such that:*

$\vec{V}_1 = \vec{\imath} + 2\vec{\jmath}$ *and* $\vec{V}_2 = 2\vec{\imath} - \vec{\jmath}$

1°/ *Find the vector sum* $\vec{S} = \vec{V}_1 + \vec{V}_2$ *, graphically and analytically.*

2°/ *Find the vector difference* $\vec{D} = \vec{V}_1 - \vec{V}_2$ *graphically and analytically.*

3°/ *The vectors* $\vec{V}_1$ *and* $\vec{V}_2$ *form a parallelogram. What represents graphically, the magnitude of the sum* $|\vec{S}|$ *and the magnitude of difference* $|\vec{D}|$ *in this parallelogram?*

4°/ *Determine the moduli of the vectors:* $\vec{V}_1, \vec{V}_2, \vec{S}$ *and* $\vec{D}$.

Additional questions: *If* $\vec{A} + \vec{B} = 5\vec{\imath} - \vec{\jmath}$ *and* $\vec{B} - \vec{A} = \vec{\imath} + \vec{\jmath}$

5°/ *Found the moduli of the vectors:* $|\vec{A}|$, $|\vec{B}|$, $|\vec{A}+\vec{B}|$ *and* $|\vec{B}-\vec{A}|$?

6°/ *Found the angles formed between:* $(\vec{A}$ *and* $\vec{B})$; $(\vec{A}+\vec{B}$ *and* $\vec{A})$; $(\vec{B}-\vec{A}$ *and* $\vec{B})$; $(\vec{A}+\vec{B}$ *and* $\vec{B}-\vec{A})$

7°/ *Determine* the *components of* $\vec{n}$ the *normal to the plane constituted by the vectors* $\vec{A}$ *and* $\vec{B}$

8°/ *What are the components of* $\vec{A}$ and $\vec{B}$ *along the directions* $\vec{u} = \vec{\imath} + \vec{\jmath}$ *and* $\vec{v} = \vec{\imath} - \vec{\jmath}$?

Exercise 02:

Given the vectors $\vec{a}$ *and* $\vec{b}$ *in an orthonormal basis,* $(\vec{\imath},\vec{\jmath},\vec{k})$ *such that:*

$$\vec{a} = 3\vec{\imath} - 5\vec{\jmath} + \vec{k} \quad \text{and} \quad \vec{b} = 2\vec{\imath} + 3\vec{\jmath} - 4\vec{k}$$

1°/ *Calculate the scalar (dot) product between* $\vec{a}$ *and* $\vec{b}$.

2°/ *What is the angle between* $\vec{a}$ *and* $\vec{b}$. *Determine* $|\vec{a}+\vec{b}|$ *and* $|\vec{a}-\vec{b}|$ *in two ways.*

3°/ *Determine the projection along the direction* $\vec{a}$ *of the vector* $\vec{b}$

If these vectors whose components are given according to the parameters α *and* β *such that*

$$\vec{a} = \alpha\vec{\imath} - 2\vec{\jmath} + \vec{k} \quad \text{and} \quad \vec{b} = \beta\vec{\imath} + \vec{\jmath} + \vec{k}$$

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4°/ *What is the relationship between* $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ *such that* $\vec{a}$ *and* $\vec{b}$ *are always perpendicular?*

Exercise 03:

Given the vectors $\vec{A}$ *and* $\vec{B}$ in an orthonormal basis $(\vec{i}, \vec{j}, \vec{k})$

$$\vec{A} = 2\vec{i} - 3\vec{j} + 4\vec{k} \qquad and \qquad \vec{A} = \vec{i} + 5\vec{j} + 2\vec{k}$$

1°/ *Calculate the vector (cross) product between* $\vec{A}$ *and* $\vec{B}$*.*

2°/ *Find the angle between* $\vec{A}$ *and* $\vec{B}$*.*

3°/ *What is the area constituted by the vectors* $\vec{A}$ *and* $\vec{B}$*.*

What is the direction of this surface?

If these vectors whose components are given according to the parameters $\boldsymbol{\gamma}$ *and* $\boldsymbol{\delta}$ *such that:*

$$\vec{A} = \gamma\vec{i} - 3\vec{j} + 4\vec{k} \qquad and \qquad \vec{B} = 5\vec{i} + \delta\vec{j} + 2\vec{k}$$

4°/ *What* are *the values of* $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ *so that* $\vec{A}$ *and* $\vec{B}$ *are always collinear?*

Exercise 04:

In an orthonormal basis $(\vec{i}, \vec{j}, \vec{k})$*, we give the vectors:*

$$\vec{A}(t) = 2t\vec{i} + (t+1)\vec{j} \qquad and \qquad \vec{B}(t) = 4t\vec{i} - 3t\vec{j} + 2\vec{k}$$

1°/ *Calculate the derivatives* $\frac{d\vec{A}}{dt}$, $\frac{d\vec{B}}{dt}$ *of the vectors* $\vec{A}$ *and* $\vec{B}$*.*

2°/ *Calculate derivatives* $\frac{d(\vec{A}\circ\vec{B})}{dt}$ *and* $\frac{d(\vec{A}\wedge\vec{B})}{dt}$ *in two ways.*

QCU:

1°/ *Let be the vectors* $\vec{A} = 3\vec{i} + 4\vec{j}$ *and* $\vec{B} = 7\vec{i} - 24\vec{j}$. *The vector having the same modulus as* $\vec{B}$ *and the same direction as* $\vec{A}$ *is:*

a/ $5\vec{i} + 20\vec{j}$ *b/* $20\vec{i} + 15\vec{j}$ *c/* $15\vec{i} + 10\vec{j}$ *d/* $15\vec{i} + 20\vec{j}$

2°/ *Let the vector* $\vec{A} = 2\vec{i} + 3\vec{j}$*. The angle between* $\vec{A}$ *and the axis* $\overline{oy}$ *is:*

a/ $arcsin\left[\frac{3}{2}\right]$ *b/* $arctg\left[\frac{3}{2}\right]$ *c/* $arctg\left[\frac{2}{3}\right]$ *d/* $arcos\left[\frac{3}{2}\right]$

3°/ 5 *forces, each equal to '* $\mathbf{10N}$ *' and applied at the same point. These forces are coplanar and angles between each two consecutive forces are same. The resultant is:*

a/ ***Zéro*** *b/* $10N$ *c/* $20N$ *d/* $10\sqrt{2}N$

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Exercise 05:

In an orthonormal basis$(\vec{i},\vec{j})$, we give the vector $\vec{A}$ such that $\vec{A} = \vec{i} + \sqrt{3}\vec{j}$

1°/ Write the unit vector $\vec{u}_A$ of $\vec{A}$ in the base.$(\vec{i},\vec{j})$

This unit vector $\vec{u}_A$ taken as a vector of the polar basis, $\vec{u}_A = \vec{u}_\rho$

2°/ Give the expression (in the Cartesian base) of the second vector of this base $\vec{u}_\theta$.

3°/ Write the vector $\vec{A}$ in the polar base.

Given a vector $\vec{B}$ in the polar basis $\vec{B} = \rho\vec{u}_\rho + \sin\theta\vec{u}_\theta$

4°/ Give the expression of $\vec{B}$ in the Cartesian base

Exercise 06:

Given a vector $\vec{A} = \vec{i} - \sqrt{3}\vec{j} - 2\vec{k}$

1°/ Give the spherical coordinates of $\vec{A}$?

2°/ What is the spherical base $(\vec{u}_r,\vec{u}_\theta,\vec{u}_\varphi)$ for $\vec{A}$, expressed in the Cartesian basis?

3°/ Do the same thing again for the vector $\vec{A}$ in the cylindrical base $(\vec{u}_\rho,\vec{u}_\theta,\vec{k})$.

Exercise 07:

In an orthonormal basis $(\vec{i},\vec{j})$, we give the point $M\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$ on the circle of radius $R = 2$ and center $C(0,0)$:

1°/ Write the unit vectors of the polar basis $(\vec{u}_\rho,\vec{u}_\theta)$ in the Cartesian basis$(\vec{i},\vec{j})$.

Given $\vec{u}_\rho$ and $\vec{u}_\theta$ for the point $M\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$.

2°/ Write the derivatives $\frac{d\vec{u}_\rho}{dt}$ and $\frac{d\vec{u}_\theta}{dt}$ of the unit vectors $\vec{u}_\rho,\vec{u}_\theta$ in the same polar basis if $\frac{d\theta}{dt} = \dot{\theta} = t$.

3°/ Write the unit vectors of the intrinsic $(\vec{u}_T,\vec{u}_N)$ basis in the Cartesian basis $(\vec{i},\vec{j})$.

Given $\vec{u}_T$ and $\vec{u}_N$ for the point $M\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$.

4°/ Write derivatives $\frac{d\vec{u}_T}{dt}$ and $\frac{d\vec{u}_N}{dt}$ of the unit vectors in the same intrinsic basis $\vec{u}_T,\vec{u}_N$.

5°/ Represent the polar and intrinsic basis at the point $M\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$

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Exercise 08: (***Additional***)

L*et a vector* $\vec{A} = 3\vec{i} + 2\vec{j} + \vec{k}$

1°/ *Give the spherical coordinates of* $\vec{A}$*?*

2°/ *Write the expressions of the spherical* base $(\vec{u}_r, \vec{u}_\theta, \vec{u}_\varphi)$**,** *in the Cartesian base*

3°/ *Do the same thing again for the vector* $\vec{A}$ *in the cylindrical base* $(\vec{u}_\rho, \vec{u}_\theta, \vec{k})$.

Exercise 09: (***HW***)

1°/ *Express the Cartesian base* $(\vec{i}, \vec{j}, \vec{k})$ *in the spherical base* $(\vec{u}_r, \vec{u}_\theta, \vec{u}_\varphi)$

2°/ *Show that the unit vectors of the spherical basis are written as follows:*

$$\frac{d\vec{u}_r}{dt} = \vec{\Omega}_1 \wedge \vec{u}_r \qquad \frac{d\vec{u}_\theta}{dt} = \vec{\Omega}_2 \wedge \vec{u}_\theta \qquad \frac{d\vec{u}_\varphi}{dt} = \vec{\Omega}_3 \wedge \vec{u}_\varphi.$$

Give the expression of $\vec{\Omega}_1$, $\vec{\Omega}_2$ *and* $\vec{\Omega}_3$**.**

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