UNIVERSITE ZIANE ACHOUR - DJELFA
FACULTE DES SCIENCES ET TECHNOLOGIES
2[imère] ANNEE LICENCE ELECTROTECHNIQUE
Module ELECTROTECHNIQUE FONDAMENTALE I

EXAMEN D'ELECTROTECHNIQUE FONDAMENTALE I

EXERCICE 1 (7 pt)

Soit le circuit suivant **Fig. 1** :

a) Calculer les courants I_1, I_2 et I_3.
b) Tracer le diagramme des phaseurs **E**, I_1, I_2, I_3 et trois tensions des branches (Vz_1, Vz_2 et Vz_3) .
c) Calculer les puissances active, réactive et apparente dans les trois branches.
d) Quelles sont les puissances réelle, réactive et apparente fournies par la source ?
e) Quel est le facteur de puissance vu par la source ?

EXERCICE 2 (7 pt) Fig. 2

On a monté en triangle, trois charges identiques résistance de **25 Ω**, inductance de **0.15 H** et capacité de **120 µF** en série sur un réseau triphasé de **tension entre lignes** de **400V - 50 Hz**.

- Déterminer les courants de ligne et de phase et la puissance active, réactive et apparente totale.

EXERCICE 3 (6 pt)

Entre les trois bornes **1, 2, 3** on dispose d'un système de tensions composées $\boldsymbol{U_{12}}$, $\boldsymbol{U_{23}}$, $\boldsymbol{U_{31}}$ formant un système triphasé déséquilibré.

Les tensions homopolaire $\boldsymbol{V_0}$, directe $\boldsymbol{V_d}$ et inverse $\boldsymbol{V_i}$ sont liées aux tensions simples par les relations :

$$\begin{cases} \boldsymbol{V_0} = \frac{1}{3}(\boldsymbol{V_a} + \boldsymbol{V_b} + \boldsymbol{V_c}) \\ \boldsymbol{V_d} = \frac{1}{3}(\boldsymbol{V_a} + \boldsymbol{a} * \boldsymbol{V_b} + \boldsymbol{a}^2 * \boldsymbol{V_c}) \\ \boldsymbol{V_i} = \frac{1}{3}(\boldsymbol{V_a} + \boldsymbol{a}^2 * \boldsymbol{V_b} + \boldsymbol{a} * \boldsymbol{V_c}) \end{cases} \quad , \boldsymbol{avec}\ \boldsymbol{a} = \boldsymbol{e}^{j\frac{2\pi}{3}} = 1\angle 120°$$

$$\begin{bmatrix} \boldsymbol{V_0} \\ \boldsymbol{V_d} \\ \boldsymbol{V_i} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{a} & \boldsymbol{a}^2 \\ 1 & \boldsymbol{a}^2 & \boldsymbol{a} \end{bmatrix} \begin{bmatrix} \boldsymbol{V_a} \\ \boldsymbol{V_b} \\ \boldsymbol{V_c} \end{bmatrix}$$

a) Calculer la quantité **:** $1 + \boldsymbol{a} + \boldsymbol{a}^2$
b) En déduire que
$$\boldsymbol{U_{12}} - \boldsymbol{a}^2\boldsymbol{U_{23}} = 3\boldsymbol{V_d}$$
$$\boldsymbol{U_{12}} - \boldsymbol{a}\boldsymbol{U_{23}} = 3\boldsymbol{V_i}$$
c) Déterminer par construction graphique des composantes $\boldsymbol{V_0}$, $\boldsymbol{V_d}$ **et** $\boldsymbol{V_i}$ **si**
$\boldsymbol{V_a} = 200\angle 0°$; $\quad \boldsymbol{V_b} = 200\angle -90° \quad \boldsymbol{V_c} = 200\angle 135°$

EXERCICE 4 (6 pt) Fig 3

Un circuit $\boldsymbol{RC}$ série $\boldsymbol{R} = 5\ \boldsymbol{\Omega}$ et $\boldsymbol{C} = 80\ \boldsymbol{mF}$, est alimenté par une tension constante $\boldsymbol{V} = 10\ \boldsymbol{V}$ à l'instant $\boldsymbol{t} = 0\ \boldsymbol{s}$ où l'interrupteur est fermé.

a) Ecrire l'équation différentielle donnant $\boldsymbol{i}$, $\boldsymbol{v_R}$ et $\boldsymbol{v_C}$
b) Calculer le courant à l'instant $\boldsymbol{t} = 1\ \boldsymbol{s}$
c) Calculer le temps au bout duquel $\boldsymbol{v_R} = \boldsymbol{v_C}$

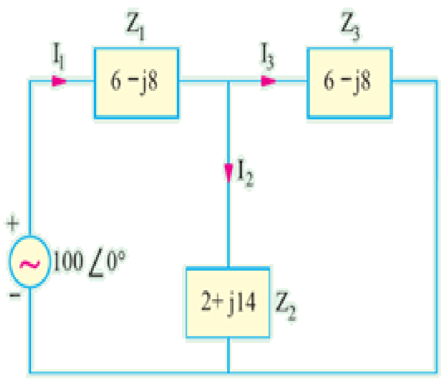

Fig1

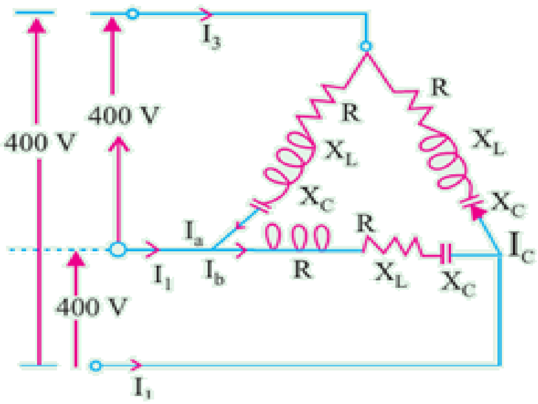

Fig 2

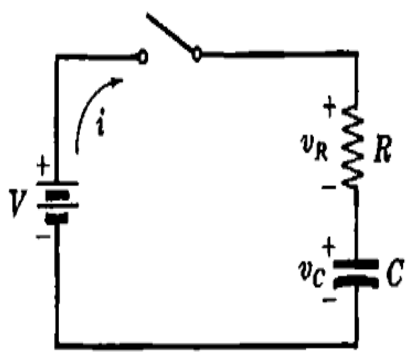

Fig 3

Solution. $Z_1 = 6 - j8 = 10 \angle 53°13°$; $Z_2 = 2 + j14 = 14.14 \angle 81.87°$; $Z_3 = 6 - j8 = 10 \angle -53.13°$

$$Z_{23} = \frac{Z_2 Z_3}{Z_2 + Z_3} = 14.14 \angle -8.13° = 14 - j2$$

Drop across two parallel impedances is given by

$$V_{23} = 100 \frac{14 - j2}{(6 - j8) + (14 - j2)} = 63.2 \angle 18.43° = 60 + j20$$

$$V_1 = 100 \frac{10 \angle -53.13°}{6 - j8 + (14 - j2)} = 47.7 \angle -26.57° = 40 - j20$$

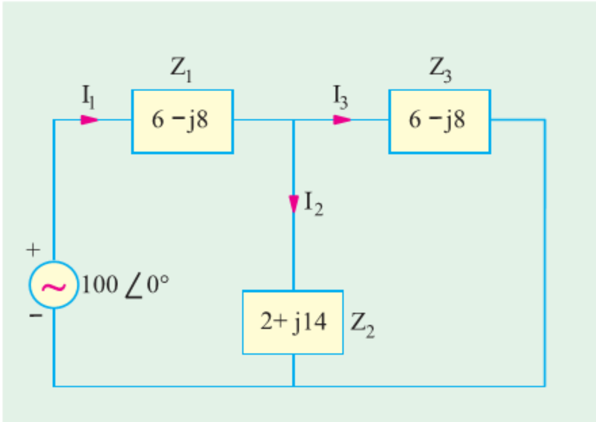

Fig. 14.41

$$I_1 = \frac{44.7 \angle -26.57°}{10 \angle -53.13°} = 4.47 \angle 26.56°$$

$$I_2 = \frac{63.2 \angle 18.43°}{14.14 \angle 81.87°} = 4.47 \angle -63.44°$$

$$I_3 = \frac{63.2 \angle 18.43°}{10 \angle -53.13°} = 6.32 \angle 71.56°$$

$$P_1 = V_1 I_1 \cos\phi_1 = 44.7 \times 4.47 \times \cos 53.13° = 120 \text{ W}$$

$$P_2 = V_2 I_2 \cos\phi_2 = 63.2 \times 4.47 \times \cos 81.87° = 40 \text{ W};$$

$$P_3 = V_3 I_3 \cos\phi_3 = 63.2 \times 6.32 \times \cos 53.13° = \mathbf{240\ W}, \text{ Total} = 400 \text{ W}$$

As a check, power delivered by the 100-V source is,

$$P = V I_1 \cos\phi = 100 \times 4.47 \times \cos 26.56° = 400 \text{ W}$$

Example 14.44. *For the circuit in Fig. 14.47 (a), given that $L = 0.159$ H*

$$C = 0.3183 \text{ mf}$$
$$I_2 = 5 \angle 60° \text{ A}$$
$$V_1 = 250 \angle 90° \text{ volts.}$$

Find :-

(i) *Impedance Z_1 with its components.*

(ii) *Source voltage in the form of $V_m \cos(\omega t + \phi)$.*

(iii) *Impedance Z_2 with its components so that source p.f. is unity, without adding to the circuit power loss.*

(iv) *Power loss in the circuit*

(v) *Draw the phasor diagram.*

(Bombay University 1997)

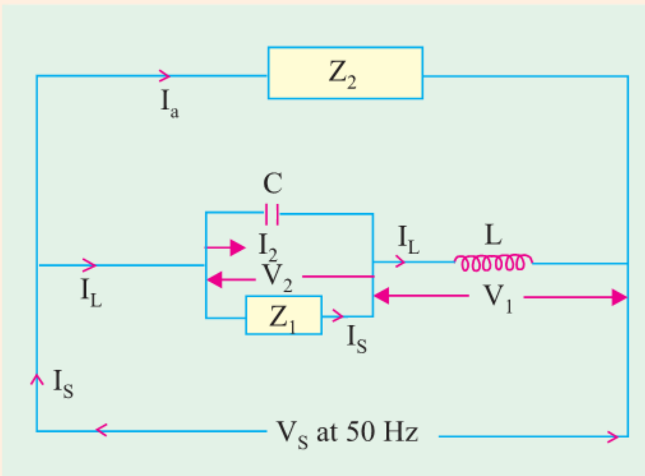

Fig. 14.47 (*a*)

Solution. $X_L = 314 \times 0.159 = 50$ ohms

$X_C = 1/(314 \times 0.3183 \times 10^{-3}) = 10$ ohms

$I_L = V_1/jX_L = (250 \angle 90^\circ) / (50 \angle 90^\circ) = 5 \angle 0^\circ$ amps

$V_2 = -jI_2 X_C = (5 \angle 60^\circ) \times (10 - \angle 90^\circ) = 50 - \angle 30^\circ$ volts $= 43.3 - j\,25$ volts

$I_L = I_L - I_2 = 5 \angle 0^\circ - 5 \angle 60^\circ = 5 + j0 - 5\,(0.5 + j0.866)$

$= 2.5 - j4.33 = 5 \measuredangle 60^\circ$

(*a*) $Z_1 = V_2/I_1 = (50 \angle -30^\circ) / 5 \measuredangle 60^\circ = 10 \angle +30^\circ$

$= 10\,(\cos 30^\circ + j \sin 30^\circ) = 8.66 + j5$

(*b*) $V_s = V_1 + V_2 = 0\,j250 + 43.3 - j25 = 43.3 + j225$

$= 229.1 \angle 79.1^\circ$ volts

V_s has a peak value of $(229.1 \times \sqrt{2} =)$ 324 volts

$V_s = 324 \cos (314\,t - 10.9^\circ)$, taking V_1 as reference

or $V_s = 325 \cos (314\,t - 79\,.1^\circ)$, taking I_L as reference.

(*c*) Source Current must be at unity P.f., with V_s

Component of I_L in phase with $V_s = 5 \cos 79.1^\circ = 0.9455$ amp

Component of I_L in quadrature with V_S (and is lagging by 90º)

$= 5.00 \times \sin 79.1^\circ = 4.91$ amp

Z_2 must carry I_a such that no power loss is there and I_S is at unity P.f. with V_s.

I_a has to be capacitive, to compensate, in magnitude, the quadrature component of I_L

$|I_a| = 4.91$ amp

$|Z_2| = V_s / |I_a| = 229.1/4.91 = 46.66$ ohms

Corresponding capacitance, $C_2 = 1 / (46.66 \times 314) = 68.34\ \mu$F

(*d*) Power-loss in the circuit $= I_1^2 \times 8.66 = 216.5$ watts or power $= V_s \times$ component of I_L in phase with $V_s = 299.1 \times 0.9455 = 216.5$ watts

(*e*) Phasor diagram is drawn in Fig. 14.47 (*b*)

Example 23.2. *Explain how an unsymmetrical system of 3-phase currents can be resolved into 3 symmetrical component systems.*

Determine the values of the symmetrical components of a system of currents

$I_R = 0 + j120A;\ I_Y = 50 - j\,100A;\ I_B = -100 - j50A$

Phase sequence is RYB. **(Elect. Engg.-I Bombay, Univ.)**

Solution. $\mathbf{I}_R \quad 0 \quad j120 \quad 120 \quad 90$

$\mathbf{I}_Y \quad 50 \quad j100 \quad 111.8 \quad 63.5\ ;\ \mathbf{I}_B \quad 100 \quad j50 \quad 111.8 \quad 153.5$

(*i*) Positive–sequence Components

$$\mathbf{I}_1 \quad \frac{1}{3}(\mathbf{I}_R \quad a\mathbf{I}_Y \quad a^2\mathbf{I}_B) \quad \frac{1}{3}\,(0 \quad j120) \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2}\,(50 \quad j100) \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2}\,(\ 100 \quad j5\ 0)$$

$= 22.8 + j108.3 = 110.7\angle 78.1° \therefore \mathbf{I}_{R1} \quad 110.7 \quad 78.1 \; ; \mathbf{I}_{Y1} \quad 110.7 \quad 41.9 \; ; \mathbf{I}_{B1} \quad 110.7 \quad 198.1$

(*ii*) Negative–sequence components

$$\mathbf{I}_2 \quad \frac{1}{3}(\mathbf{I}_R \quad a^2\mathbf{I}_Y \quad a\mathbf{I}_B) \quad \frac{1}{3}(\; 18.3 \quad j65.1) \quad 6.1 \quad j21.7 \quad 22.5 \quad 105.7$$

$\therefore \; \mathbf{I}_{R2} \quad 22.5 \quad 105.7 \; ; \; \mathbf{I}_{Y2} \quad 22.5 \quad 225 \; ; \; \mathbf{I}_{B2} \quad 22.5 \quad 14.3$

(*iii*) Zero-sequence component

$$\mathbf{I}_0 \quad \frac{1}{3}(\mathbf{I}_R \quad \mathbf{I}_Y \quad \mathbf{I}_B) \quad \frac{1}{3}[(0 \quad j120) \quad (50 \quad j100) \quad (\; 100 \quad j50)] \quad 16.7 \quad j10$$

As a check, it may be found that

$$\mathbf{I}_R = \mathbf{I}_{R1} + \mathbf{I}_{R2} + \mathbf{I}_0 ; \mathbf{I}_Y = \mathbf{I}_{Y1} + \mathbf{I}_{Y2} + \mathbf{I}_0 ; \mathbf{I}_{B1} + \mathbf{I}_{B2} + \mathbf{I}_0$$

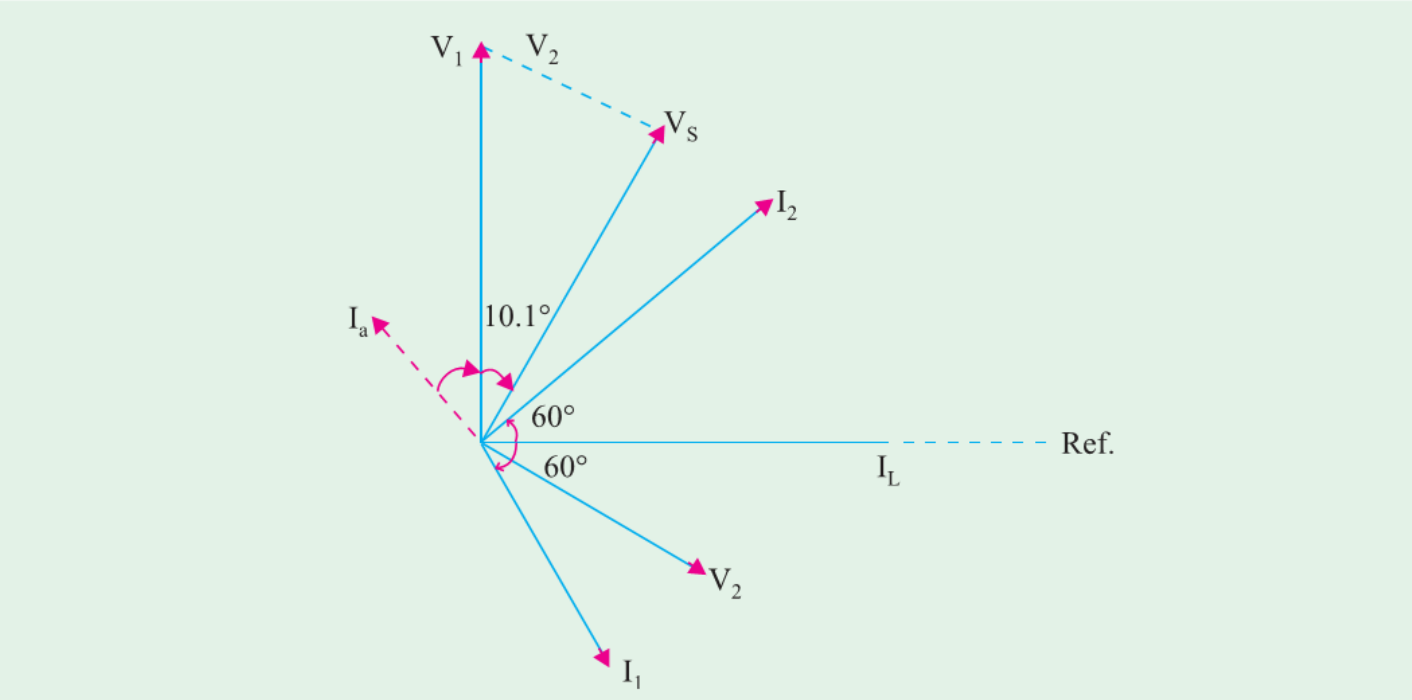

Fig. 14.47 (*b*) Phasor diagram

Example 19.10. *A three pahse 400-V, 50 Hz, a.c. supply is feeding a three phase delta-connected load with each phase having a resistance of 25 ohms, an inductance of 0.15 H, and a capacitor of 120 microfarads in series. Determine the line current, volt-amp, active power and reactive volt-amp.* **[Nagpur University, November 1999]**

EXERCICE 2

On a monté en triangle, trois charges identiques résistance de 25 Ω, inductance de 0.15 H et capacité de 120 µF en série sur un réseau triphasé de tension entre lignes de 400V - 50 Hz

Déterminer le courant de ligne et de phase et les puissance actives, réactive et apparente

Solution. Impedance per phase $r + jX_L - jX_C$

$$X_L = 2\pi \times 50 \times 0.15 = 47.1\ \Omega$$

$$X_C = \frac{10^6}{32{\cdot}37} = 26.54\ \Omega$$

$\cos\phi = \dfrac{25}{32.37}$ Lagging, since inductive reactance is dominating.

$$\text{Phase Current} = \frac{400}{25 + j20.56} = 12.357$$

Line Current = $\sqrt{3} \times 12.357$ = 21.4 amp

Since the power factor is 0.772 lagging,

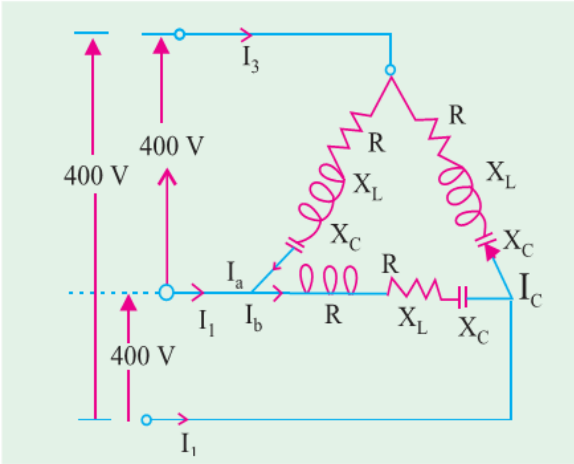

Fig. 19.22

P = total three phase power = $\sqrt{3}\, V_L\ I_L\ \cos\phi \times 10^{-3}$ kW

= $\sqrt{3}$ × 400 × 21.4 × 0.772 × 10^{-3} = 11.446 kW

S = total 3 ph kVA $\dfrac{11.446}{0.772}$ = 14.83 kVA 14.83 kVA

Q = total 3 ph "reactive kilo-volt-amp" $\sqrt{3}$ = $(S^2 - P^2)^{0.50}$ = 9.43 kVAR lagging