Physics 454 Math Formulas, 5 February 2003

Cartesian coordinates:

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

$$\nabla \times \vec{V} = \left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right] + \left[\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right] + \left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right] \qquad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical coordinates:

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial}{\partial \phi}(V_\phi) + \frac{\partial}{\partial z}(V_z)$$

$$\nabla \times \vec{V} = \left[\frac{1}{r}\frac{\partial}{\partial \phi}(V_z) - \frac{\partial}{\partial z}(V_\phi)\right]\hat{r} + \left[\frac{\partial}{\partial z}(V_r) - \frac{\partial}{\partial r}(V_z)\right]\hat{\phi} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rV_\phi) - \frac{\partial}{\partial \phi}(V_r)\right]\hat{z}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical coordinates:

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi} \qquad \nabla \cdot \vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta V_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(V_\phi)$$

$$\nabla \times \vec{V} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta V_\phi) - \frac{\partial}{\partial \phi}(V_\theta)\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial \phi}(V_r) - \frac{\partial}{\partial r}(rV_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial}{\partial \theta}(V_r)\right]\hat{\phi}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^{2f}}{\partial \phi^2}$$

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla\left(\vec{A}\cdot\vec{B}\right) = \vec{A}\times\left(\nabla\times\vec{B}\right) + \vec{B}\times\left(\nabla\times\vec{A}\right) + \left(\vec{A}\cdot\nabla\right)\vec{B} + \left(\vec{B}\cdot\nabla\right)\vec{A}$$

$$\nabla\cdot\left(f\vec{A}\right) = f\left(\nabla\cdot\vec{A}\right) + \vec{A}\cdot(\nabla f)$$

$$\nabla\cdot\left(\vec{A}\times\vec{B}\right) = \vec{B}\cdot\left(\nabla\times\vec{A}\right) - \vec{A}\cdot\left(\nabla\times\vec{B}\right)$$

$$\nabla\times\left(f\vec{A}\right) = f\left(\nabla\times\vec{A}\right) + \vec{A}\times(\nabla f)$$

$$\nabla\times\left(\vec{A}\times\vec{B}\right) = \left(\vec{B}\cdot\nabla\right)\vec{A} - \left(\vec{A}\cdot\nabla\right)\vec{B} + \vec{A}\left(\nabla\cdot\vec{B}\right) - \vec{B}\left(\nabla\cdot\vec{A}\right)$$

$$\vec{A}\times\vec{B} = \det\begin{bmatrix}\hat{x} & \hat{y} & \hat{z}\\ A_x & A_y & A_z \\ B_x & B_y & B_z\end{bmatrix} = \left(A_yB_z - S_zB_y\right)\hat{x} + \left(A_zB_x - A_xB_z\right)\hat{y} + \left(A_xB_y - A_yB_x\right)\hat{z}$$

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \vec{B}\cdot\left(\vec{C}\times\vec{A}\right) = \vec{C}\cdot\left(\vec{A}\times\vec{B}\right)$$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{B}\left(\vec{A}\cdot\vec{C}\right) - \vec{C}\left(\vec{A}\cdot\vec{B}\right) \qquad \left(\vec{A}\times\vec{B}\right)\times\vec{C} = \vec{B}\left(\vec{C}\cdot\vec{A}\right) - \vec{A}\left(\vec{C}\cdot\vec{B}\right)$$

$$\delta_{ij} = \begin{cases} +1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases} \qquad \varepsilon_{ijk} = \begin{cases} 0 \text{ if } i = j\,,\ j = k\,,\text{ or } k = i \\ +1 \text{ if } \boldsymbol{ijk} = 123,\ 231,\ 312 \\ -1 \text{ if } \boldsymbol{ijk} = 321,\ 213,\ 132 \end{cases}$$

$$\partial_i = \frac{\partial}{\partial x_i} \qquad \varepsilon_{ijk}\varepsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

$$\vec{\mathrm{A}}\cdot\vec{\mathrm{B}} = \mathrm{A}_i\mathrm{B}_i = A_i\delta_{ij}B_j \qquad \vec{A}\times\vec{B}\Big|_k = \varepsilon_{ijk}A_iB_j \qquad \vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \varepsilon_{ijk}A_iB_jC_k$$

$$\left[\vec{A}\times\left(\vec{B}\times\vec{C}\right)\right]_k = B_kA_iC_i - C_kA_iB_i$$

$$\left[\nabla(fg)\right]_k = \partial_k(fg) = f\,\partial_k g + g\partial_k f$$

$$\left[\nabla\left(\vec{A}\cdot\vec{B}\right)\right]_k = \partial_k\left(A_iB_i\right) = B_i\partial_kA_i + A_i\partial_kB_i$$

$$\nabla\cdot\left(f\vec{A}\right) = \partial_i\left(f A_i\right) = f\,\partial_iA_i + A_i\partial_i f$$

$$\nabla\cdot\left(\vec{A}\times\vec{B}\right) = \varepsilon_{ijk}\partial_i\left(A_jB_k\right) = \varepsilon_{ijk}B_k\partial_iA_j + \varepsilon_{ijk}A_j\partial_iB_k$$

$$\left[\nabla\times\left(f\vec{A}\right)\right]_k = \varepsilon_{ijk}\partial_i\left(f A_j\right) = \varepsilon_{ijk}f\,\partial_iA_j + \varepsilon_{ijk}A_j\partial_i f$$

$$\left[\nabla\times\left(\vec{A}\times\vec{B}\right)\right]_k = \varepsilon_{ijk}\partial_i\varepsilon_{lmj}A_lB_m = \varepsilon_{kij}\varepsilon_{lmj}\partial_iA_lB_m = \left(\delta_{kl}\delta_{im} - \delta_{km}\delta_{il}\right)\partial_iA_lB_m$$

$$= \partial_iA_kB_i - \partial_iA_iB_k = B_i\partial_iA_k + A_k\partial_iB_i - B_k\partial_iA_i - A_i\partial_iB_k$$

10^{-12} = pico 10^{-9} = nano 10^{-6} = micro (μ) 10^{-3} = milli

10^{+3} = kilo 10^{+6} = Mega 10^{+9} = Giga 10^{+12} = Tera