CORRIGE TYPE DE L'EXAMEM

Exercice N°1 (7 pts)

1. Entropie H:

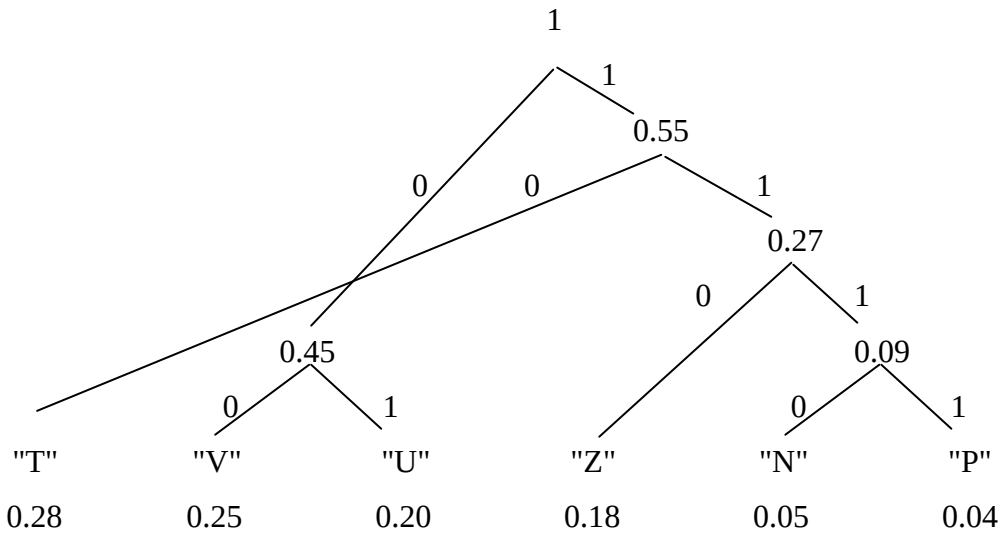
H=p(T)*log2(1/p(T))+p(V)*log2(1/p(V))+p(U)*log2(1/p(U))+p(Z)*log2(1/p(Z))+p(N)*log2(1/p(N))+ p(P)*log2(1/p(P))

H=0.28*log2(1/0.28)+0.25*log2(1/0.25)+0.20*log2(1/0.20)+0.18*log2(1/0.18)+0.05*log2(1/0.05) + 0.04*log2(1/0.04)

H=2.32 bits/symbole (1 pt)

2. Code de Huffman :

Arbre : **(1 pt)**



"T" : **10**, n(T)=2. **(0.5 pt)**

"V" : **00**, n(V)=2. **(0.5 pt)**

"U" : **01**, n(U)=2. **(0.5 pt)**

"Z" : **110**, n(Z)=3. **(0.5 pt)**

"N" : **1110**, n(N)=4. **(0.5 pt)**

"P" : **1111**, n(P)=4. **(0.5 pt)**

3. Longueur moyenne L :

H=p(T)*n(T)+ p(V)*n(V)+ p(U)*n(U)+ p(Z)*n(Z)+ p(N)*n(N)+ p(P)*n(P)

L=0.28*2+0.25*2+0.2*2+0.18*3+0.05*4+0.04*4

L=2.36 bits/symbole (1 pt)

4. Efficacité E:

E=H/L

E=2.32/2.36

E=98.31% (1 pt)

Exercice N°2 (6 pts)

1. les mots de codes sont :

0000000 - 1110010 - 0111001 - 1001011 - 1100101 - 0010111 - 1011100 - 0101110. (2 pts)

2. La distance du code est d: **d=4 (1 pt)**

3. Nombre d'erreurs à détecter ed=d-1: **ed=3 (0.5 pt)**

Nombre d'erreurs à corriger ec=(d-1)/2: **ec=1 (0.5 pt)**

$$G=\begin{bmatrix} a \\ b \\ c \end{bmatrix}=\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\boldsymbol{G=\begin{bmatrix} a+b \\ a+b+c \\ a+c \end{bmatrix}=\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}}$$ **(1 pt)**

Le polynôme générateur est : **$g(x)=x^4+x^2+x+1$ (1 pt)**

Exercice N°3 (7 pts)

$$C1=\left\{\begin{bmatrix} 0 & 4 \\ 2 & 5 \end{bmatrix}\equiv mat1,\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}\equiv mat2\right\},\ M1=\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$ **(0.5 pt)**

$$C2=\left\{\begin{bmatrix} 6 & 2 \\ 0 & 1 \end{bmatrix}\equiv mat3,\begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}\equiv mat4\right\},\ M2=\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$ **(0.5 pt)**

$\text{dist(mat1,M1)}=\sqrt{(0-2)^2+(4-2)^2+(2-1)^2+(5-3)^2}=3.60$

$\text{dist(mat1,M2)}=\sqrt{(0-4)^2+(4-3)^2+(2-1)^2+(3-2)^2}=4.35$ **(0.5 pt)**

$\text{dist(mat1,M1)}<\text{dist(mat1,M2)}\Rightarrow \text{mat1}\in C1$

$\text{dist(mat2,M1)}=\sqrt{(4-2)^2+(0-2)^2+(0-1)^2+(1-3)^2}=3.60$

$\text{dist(mat2,M2)}=\sqrt{(4-4)^2+(0-3)^2+(0-1)^2+(1-2)^2}=3.31$ **(0.5 pt)**

$\text{dist(mat2,M2)}<\text{dist(mat2,M1)}\Rightarrow \text{mat2}\in C2$

$\text{dist(mat3,M1)}=\sqrt{(6-2)^2+(2-2)^2+(0-1)^2+(1-3)^2}=3$

$\text{dist(mat3,M2)}=\sqrt{(6-4)^2+(2-3)^2+(0-1)^2+(1-2)^2}=2.64$ **(0.5 pt)**

$\text{dist(mat3,M2)}<\text{dist(mat3,M1)}\Rightarrow \text{mat3}\in C2$

$\text{dist(mat4,M1)}=\sqrt{(2-2)^2+(4-2)^2+(2-1)^2+(3-3)^2}=2.23$

$\text{dist(mat4,M2)}=\sqrt{(2-4)^2+(4-3)^2+(2-1)^2+(3-2)^2}=2.64$ **(0.5 pt)**

$\text{dist(mat4,M1)}<\text{dist(mat3,M2)}\Rightarrow \text{mat4}\in C1$

C1 = $\left\{\begin{bmatrix}0 & 4\\ 2 & 5\end{bmatrix} \equiv mat1, \begin{bmatrix}2 & 4\\ 2 & 3\end{bmatrix} \equiv mat4\right\}$, M1=$\begin{bmatrix}1 & 4\\ 2 & 4\end{bmatrix}$

C2 = $\left\{\begin{bmatrix}6 & 2\\ 0 & 1\end{bmatrix} \equiv mat3, \begin{bmatrix}4 & 0\\ 0 & 1\end{bmatrix} \equiv mat2\right\}$, M2=$\begin{bmatrix}5 & 1\\ 0 & 1\end{bmatrix}$

dist(mat1,M1)= $\sqrt{(0-1)^2+(4-4)^2+(2-2)^2+(5-4)^2} = 1.41$

dist(mat1,M2)= $\sqrt{(0-5)^2+(4-1)^2+(2-0)^2+(3-1)^2} = 6.48$ **(0.5 pt)**

dist(mat1,M1) <dist(mat1,M2) $\Rightarrow$ mat1$\in$C1

dist(mat4,M1)= $\sqrt{(2-1)^2+(4-4)^2+(2-2)^2+(3-4)^2} = 1.41$

dist(mat4,M2)= $\sqrt{(2-5)^2+(4-1)^2+(2-0)^2+(3-1)^2} = 5.09$ **(0.5 pt)**

dist(mat4,M1) <dist(mat4,M2) $\Rightarrow$ mat4$\in$C1

dist(mat3,M1)= $\sqrt{(6-1)^2+(2-4)^2+(0-2)^2+(1-4)^2} = 6.48$

dist(mat3,M2)= $\sqrt{(6-5)^2+(2-1)^2+(0-0)^2+(1-1)^2} = 1.41$ **(0.5 pt)**

dist(mat3,M2) <dist(mat3,M1) $\Rightarrow$ mat3$\in$C2

dist(mat2,M1)= $\sqrt{(4-1)^2+(0-4)^2+(0-2)^2+(1-4)^2} = 6.92$

dist(mat2,M2)= $\sqrt{(4-5)^2+(0-1)^2+(0-0)^2+(1-1)^2} = 1.41$ **(0.5 pt)**

dist(mat2,M2) <dist(mat2,M1) $\Rightarrow$ mat2$\in$C2

Donc :

C1 = $\left\{\begin{bmatrix}\mathbf{0} & \mathbf{4}\\ \mathbf{2} & \mathbf{5}\end{bmatrix} \equiv \boldsymbol{mat}\,\mathbf{1}, \begin{bmatrix}\mathbf{2} & \mathbf{4}\\ \mathbf{2} & \mathbf{3}\end{bmatrix} \equiv \boldsymbol{mat}\,\mathbf{4}\right\}$**, barycentre M1=**$\begin{bmatrix}\mathbf{1} & \mathbf{4}\\ \mathbf{2} & \mathbf{4}\end{bmatrix}$ **(1 pt)**

C2 = $\left\{\begin{bmatrix}\mathbf{6} & \mathbf{2}\\ \mathbf{0} & \mathbf{1}\end{bmatrix} \equiv \boldsymbol{mat}\,\mathbf{3}, \begin{bmatrix}\mathbf{4} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{bmatrix} \equiv \boldsymbol{mat}\,\mathbf{2}\right\}$**, barycentre M2=**$\begin{bmatrix}\mathbf{5} & \mathbf{1}\\ \mathbf{0} & \mathbf{1}\end{bmatrix}$ **(1 pt)**