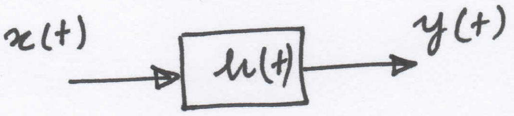
Série TD N° 03

***Exo1 :**



$$y(t) = \int_{-\infty}^{+\infty} x(t-\tau)\, h(\tau)\, d\tau .$$

$$y(t) = x(t) * h(t).$$

1)° si $x(t) = \delta(t)$

$$\Rightarrow y(t) = \int_{-\infty}^{+\infty} \delta(t-\tau)\, h(\tau)\, d\tau .$$

$\boxed{y(t) = h(t)}$ La sotie st la même que le systeme $h(t)$.

2)° si $h(t) = \delta(t)$

$$\Rightarrow y(t) = \int_{-\infty}^{+\infty} x(t-\tau)\, \delta(\tau)\, d\tau .$$

$\boxed{y(t) = x(t)}$ La sotie st la même que l'entrée

(on peut utiliser la TF : $y(t) = x(t) * h(t) \xrightarrow{TF} Y(f) = X(f) \cdot H(f)$.

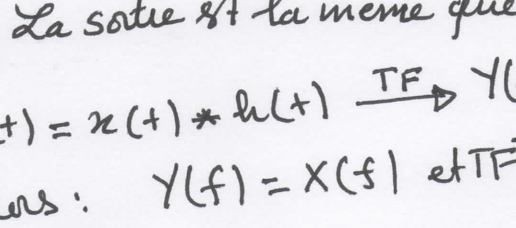
comme $h(t) = \delta(t) \xrightarrow{TF} 1$ alors : $Y(f) = X(f)$ et $TF^{-1}\{Y(f)\} = y(t) = x(t)$)

3)° si $h(t) = \mathcal{E}(t)$ $\qquad \mathcal{E}(t) = \begin{cases} 1 \; ; t \geqslant 0 \\ 0 \; \sim . \end{cases}$

$$\Rightarrow y(t) = \int_{-\infty}^{+\infty} x(t-\tau)\, h(\tau)\, d\tau = \int_{0}^{+\infty} x(t-\tau)\, 1(\tau)\, d\tau .$$

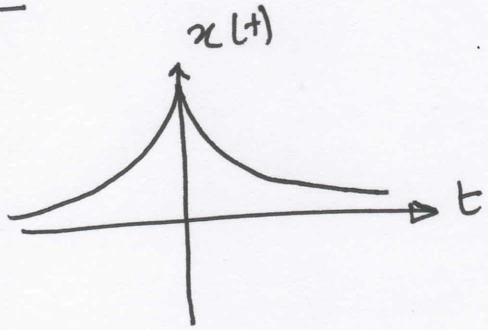
$$y(t) = \int_{0}^{+\infty} x(t-\tau)\, d\tau .$$

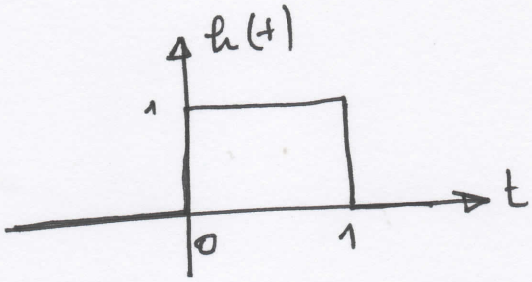
Donc la sotie st l'integral de l'entrée (integrateur)



* Exo2 :

1)°





$$x(t) = \begin{cases} e^{t} & ; t < 0 \\ e^{-t} & ; t \geq 0 \end{cases}$$

$$h(t) = \mathrm{Rect}\left(t - \tfrac{1}{2}\right).$$

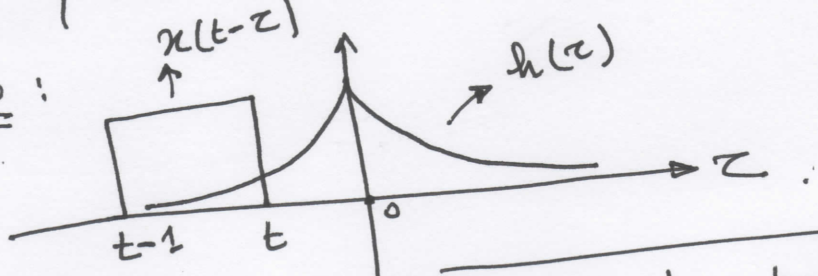
$$h(t) = \begin{cases} 1 & ; 0 \leq t \leq 1 \\ 0 & \sim \end{cases}$$

2)° Le produit de convolution :

$$y(t) = x(t) * h(t) \quad (\text{commutatif}).$$
$$= h(t) * x(t)$$

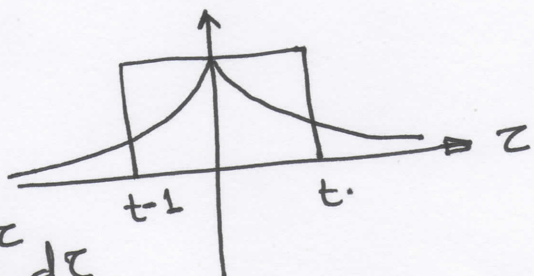
Alors : on prend : $\begin{cases} h(t) \longrightarrow \text{fixe}. \\ x(t) \longrightarrow \text{se deplace}. \end{cases}$

1)^{er} cas : $t < 0$:



$$y(t) = \int_{t-1}^{t} e^{\tau} d\tau = \left[e^{\tau}\right]_{t-1}^{t} \Rightarrow \boxed{y(t) = e^{t} - e^{t-1}}$$

2)^{e} cas : $0 < t < 1$:



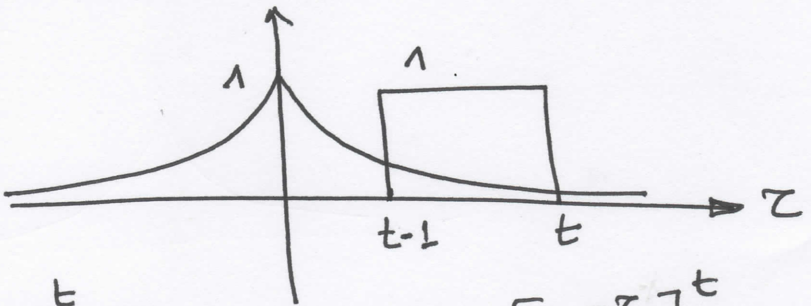
$$y(t) = \int_{t-1}^{0} e^{\tau} d\tau + \int_{0}^{t} e^{-\tau} d\tau$$

$$y(t) = \left[e^{\tau}\right]_{t-1}^{0} - \left[e^{-\tau}\right]_{0}^{t}$$

$$= 1 - e^{t-1} - e^{-t} + 1$$

$$\boxed{y(t) = 2 - e^{-t} - e^{t-1}}$$

3)° cas : $t-1>0 \Rightarrow t>1$



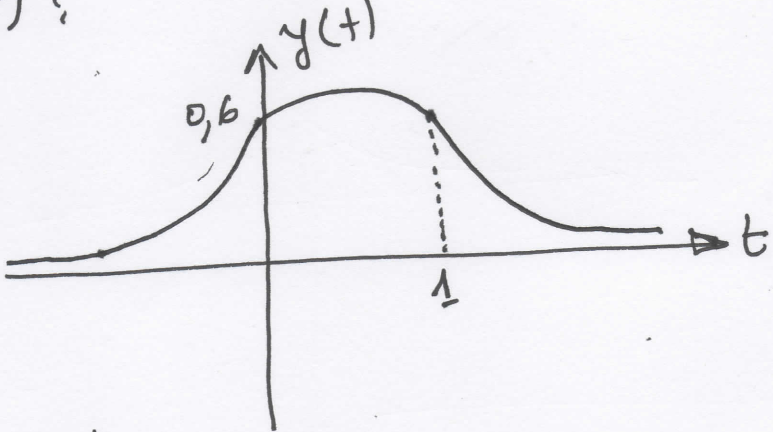
$$y(t) = \int_{t-1}^{t} e^{-\tau}\, d\tau \Rightarrow y(t) = \left[-e^{-\tau}\right]_{t-1}^{t}$$

$$\Rightarrow \boxed{y(t) = e^{-t+1} - e^{-t}}$$

Alors,

$$y(t) = \begin{cases} e^{t} - e^{t-1} & ; \ t<0 \\ 2 - e^{-t} - e^{t-1} & ; \ 0<t<1 \\ e^{-t+1} - e^{-t} & ; \ t>1 . \end{cases}$$

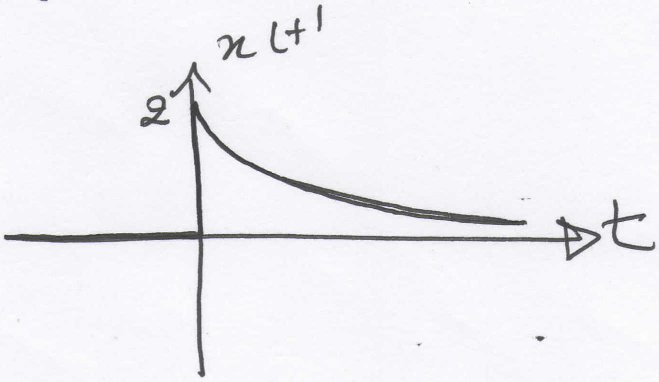
3)° Allure de $y(t)$:



* exo3 :

$$x(t) = \begin{cases} 2e^{-t} & ; \ t \geq 0 \\ 0 & \sim . \end{cases}$$

1)



2)° Calcul de la TF:

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt.$$

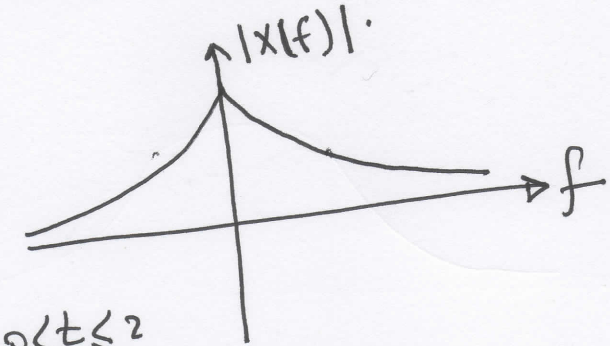
$$= \int_{0}^{+\infty} 2 e^{-t} e^{-j2\pi f t}\, dt = 2\int_{0}^{+\infty} e^{-(1+2j\pi f)t}\, dt.$$

$$X(f) = \frac{-2}{1+j2\pi f}\left[e^{-(1+j2\pi f)t}\right]_0^{+\infty}$$

$$\Rightarrow \boxed{X(f) = \frac{2}{1+j2\pi f}}$$

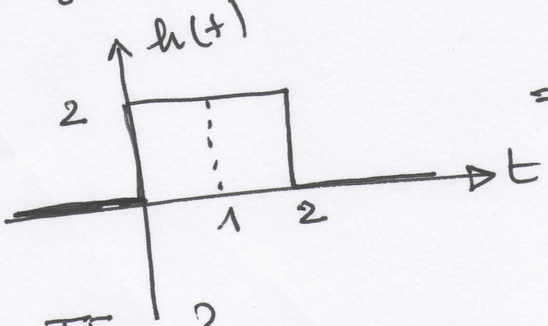
3)° Spectre en amplitude : $|X(f)|$:

$$|X(f)| = \left|\frac{2}{1+j2\pi f}\right| = \frac{|2|}{|1+j2\pi f|} = \frac{2}{\sqrt{1+4\pi^2 f^2}}.$$



4) $h(t) = \begin{cases} 2 & ; 0 \leq t \leq 2 \\ 0 & \sim \end{cases}$

a)



$\Rightarrow 2\,\mathrm{Rect}\left(\frac{t-1}{2}\right)$.

b) $2\,\mathrm{Rect}\left(\frac{t-1}{2}\right) \xrightarrow{TF} ?$

on utilise d'abord la propriété : $x(t-t_0) \xrightarrow{TF} e^{-j2\pi f t_0} X(f)$

et $A\,\mathrm{Rect}\left(\frac{t}{T}\right) \xrightarrow{TF} AT\,\mathrm{sinc}(\pi f T)$

$X(f) = AT\,\text{sinc}(\pi f T)$ avec $A = 2$ et $T = 2$.

$X(f) = 4\,\text{Sinc}(2\pi f)$.

Alors :

$$H(f) = e^{-j2\pi f \overset{t_0}{(1)}} \cdot 4\,\text{Sinc}(2\pi f).$$

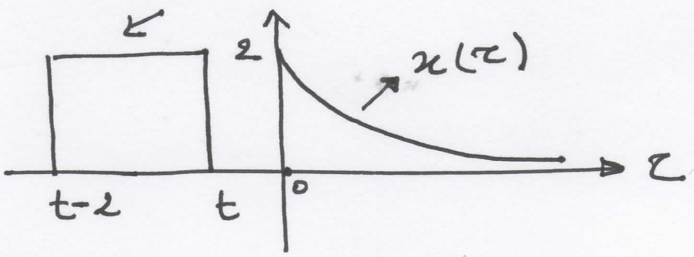
$$\Rightarrow \boxed{H(f) = 4\,e^{-j2\pi f}\,\text{sinc}(2\pi f)}$$

c) Le produit de convolution :

$$y(t) = x(t) * h(t).$$

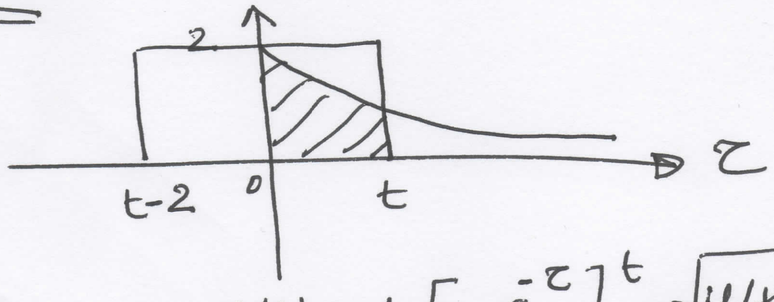
$$y(t) = \int_{-\infty}^{+\infty} h(t-\tau)\, x(\tau)\, d\tau. \longrightarrow \begin{cases} x : \text{fixe} \\ h : \text{se deplace}. \end{cases}$$

1)er cas : $t<0$



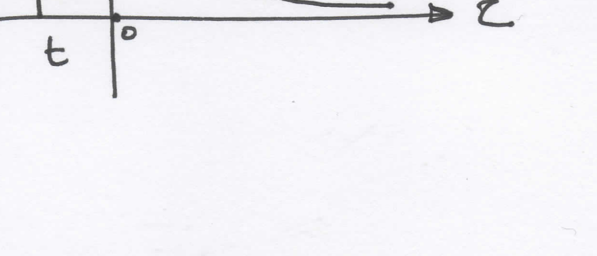
$$\boxed{y(t) = 0}$$

2)e cas : $0<t<2$:



$$y(t) = \int_0^t 4e^{-\tau}\,d\tau \Rightarrow y(t) = 4\left[-e^{-\tau}\right]_0^t \Rightarrow \boxed{y(t) = 4(1-e^{-t})}$$

3)e cas : $t>2$



$$y(t) = \int_{t-2}^{t} 4\,e^{-\tau}\,d\tau$$

$$\Rightarrow y(t) = \boxed{4\left(-e^{-t} + e^{-t+2}\right)}$$

d) TF de $y(t)$

$$y(t) = x(t) * h(t) \xrightarrow{TF} Y(f) = X(f) \cdot H(f).$$

$$Y(f) = \frac{2}{1+j2\pi f} \cdot 4 e^{-j2\pi f} \operatorname{sinc}(2\pi f).$$

$$\boxed{Y(f) = \frac{8 e^{-j2\pi f}}{1+j2\pi f} \operatorname{sinc}(2\pi f)}$$