2-7. A 5000-kVA 230/13.8-kV single-phase power transformer has a per-unit resistance of 1 percent and a per-unit reactance of 5 percent (data taken from the transformer's nameplate). The open-circuit test performed on the low-voltage side of the transformer yielded the following data:

$V_{OC} = 13.8 \text{ kV} \qquad I_{OC} = 15.1 \text{ A} \qquad P_{OC} = 44.9 \text{ kW}$

(a) Find the equivalent circuit referred to the low-voltage side of this transformer.

(b) If the voltage on the secondary side is 13.8 kV and the power supplied is 4000 kW at 0.8 PF lagging, find the voltage regulation of the transformer. Find its efficiency.

SOLUTION

(a) The open-circuit test was performed on the low-voltage side of the transformer, so it can be used to directly find the components of the excitation branch relative to the low-voltage side.

$$|Y_{EX}| = |G_C - jB_M| = \frac{15.1 \text{ A}}{13.8 \text{ kV}} = 0.0010942$$

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} = \cos^{-1}\frac{44.9 \text{ kW}}{(13.8 \text{ kV})(15.1 \text{ A})} = 77.56°$$

$$Y_{EX} = G_C - jB_M = 0.0010942\angle -77.56° \text{ S} = 0.0002358 - j0.0010685 \text{ S}$$

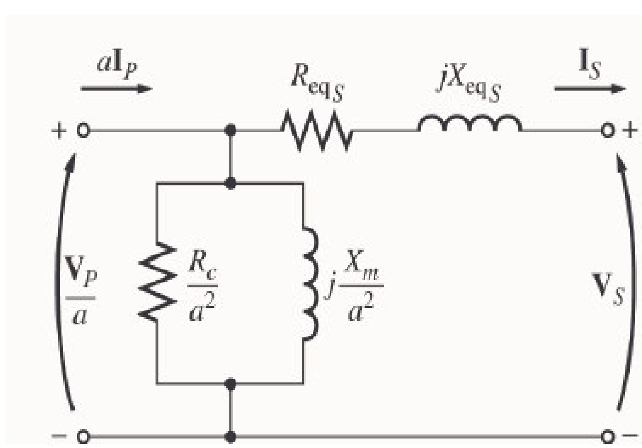
$$R_C = \frac{1}{G_C} = 4240\ \Omega$$

$$X_M = \frac{1}{B_M} = 936\ \Omega$$

The base impedance of this transformer referred to the secondary side is

$$Z_{base} = \frac{V_{base}^{\ 2}}{S_{base}} = \frac{(13.8 \text{ kV})^2}{5000 \text{ kVA}} = 38.09\ \Omega$$

so $R_{EQ} = (0.01)(38.09\ \Omega) = 0.38\ \Omega$ and $X_{EQ} = (0.05)(38.09\ \Omega) = 1.9\ \Omega$. The resulting equivalent circuit is shown below:



$$R_{EQ,s} = 0.38\,\Omega \qquad X_{EQ,s} = j1.9\,\Omega$$

$$R_{C,s} = 4240\,\Omega \qquad X_{M,s} = 936\,\Omega$$

(b) If the load on the secondary side of the transformer is 4000 kW at 0.8 PF lagging and the secondary voltage is 13.8 kV, the secondary current is

$$I_S = \frac{P_{\text{LOAD}}}{V_S\ \text{PF}} = \frac{4000\text{ kW}}{(13.8\text{ kV})(0.8)} = 362.3\text{ A}$$

$$\mathbf{I}_S = 362.3\angle -36.87^\circ\text{ A}$$

The voltage on the primary side of the transformer (referred to the secondary side) is

$$\mathbf{V}_P{}' = \mathbf{V}_S + \mathbf{I}_S Z_{EQ}$$

$$\mathbf{V}_P{}' = 13{,}800\angle 0^\circ\text{ V} + (362.3\angle -36.87^\circ\text{ A})(0.38 + j1.9\ \Omega) = 14{,}330\angle 1.9^\circ\text{ V}$$

There is a voltage drop of 14 V under these load conditions. Therefore the voltage regulation of the transformer is

$$\text{VR} = \frac{14{,}330 - 13{,}800}{13{,}800} \times 100\% = 3.84\%$$

The transformer copper losses and core losses are

$$P_{\text{CU}} = I_S{}^2\, R_{EQ,S} = (362.3\text{ A})^2 (0.38\ \Omega) = 49.9\text{ kW}$$

$$P_{\text{core}} = \frac{\left(V_P{}'\right)^2}{R_C} = \frac{(14{,}330\text{ V})^2}{4240\ \Omega} = 48.4\text{ kW}$$

Therefore the efficiency of this transformer at these conditions is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}}} \times 100\% = \frac{4000\text{ kW}}{4000\text{ kW} + 49.9\text{ kW} + 48.4\text{ kW}} = 97.6\%$$

2-10. A 13,800/480 V three-phase Y-Δ-connected transformer bank consists of three identical 100-kVA 7967/480-V transformers. It is supplied with power directly from a large constant-voltage bus. In the short-circuit test, the recorded values on the high-voltage side for one of these transformers are

$$V_{SC} = 560 \text{ V} \qquad I_{SC} = 12.6 \text{ A} \qquad P_{SC} = 3300 \text{ W}$$

(a) If this bank delivers a rated load at 0.85 PF lagging and rated voltage, what is the line-to-line voltage on the primary of the transformer bank?

(b) What is the voltage regulation under these conditions?

SOLUTION From the short-circuit information, it is possible to determine the per-phase impedance of the transformer bank referred to the high-voltage side. The primary side of this transformer is Y-connected, so the short-circuit phase voltage is

$$V_{\phi,SC} = \frac{V_{SC}}{\sqrt{3}} = \frac{560 \text{ V}}{\sqrt{3}} = 323.3 \text{ V}$$

the short-circuit phase current is

$$I_{\phi,SC} = I_{SC} = 12.6 \text{ A}$$

and the power per phase is

$$P_{\phi,SC} = \frac{P_{SC}}{3} = 1100 \text{ W}$$

Thus the per-phase impedance is

$$\left|Z_{EQ}\right| = \left|R_{EQ} + jX_{EQ}\right| = \frac{323.3 \text{ V}}{12.6 \text{ A}} = 25.66 \ \Omega$$

$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{1100 \text{ W}}{(323.3 \text{ V})(12.6 \text{ A})} = 74.3°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 25.66\angle 74.3° \ \Omega = 6.94 + j24.7 \ \Omega$$

$$R_{EQ} = 6.94 \ \Omega$$

$$X_{EQ} = j24.7 \ \Omega$$

(a) If this Y-Δ transformer bank delivers rated kVA (300 kVA) at 0.85 power factor lagging while the secondary voltage is at rated value, then each transformer delivers 100 kVA at a voltage of 480 V and 0.85 PF lagging. Referred to the *primary side* of one of the transformers, the load on each transformer is equivalent to 100 kVA at 7967 V and 0.85 PF lagging. The equivalent current flowing in the secondary of one transformer referred to the primary side is

$$I_{\phi,S}' = \frac{100\ \text{kVA}}{7967\ \text{V}} = 12.55\ \text{A}$$

$$\mathbf{I}_{\phi,S}' = 12.55\angle -31.79^\circ\ \text{A}$$

The voltage on the primary side of a single transformer is thus

$$\mathbf{V}_{\phi,P} = \mathbf{V}_{\phi,S}' + \mathbf{I}_{\phi,S}' Z_{\text{EQ},P}$$

$$\mathbf{V}_{\phi,P} = 7967\angle 0^\circ\ \text{V} + (12.55\angle -31.79^\circ\ \text{A})(6.94 + j24.7\ \Omega) = 8207\angle 1.52^\circ\ \text{V}$$

The line-to-line voltage on the primary of the transformer is

$$V_{\text{LL},P} = \sqrt{3}\ V_{\phi,P} = \sqrt{3}(8207\ \text{V}) = 14.22\ \text{kV}$$

(b) The voltage regulation of the transformer is

$$\text{VR} = \frac{8207\text{-}7967}{7967} \times 100\% = 3.01\%$$

Note: It is much easier to solve problems of this sort in the per-unit system, as we shall see in the next problem.

2-22. A single-phase 10-kVA 480/120-V transformer is to be used as an autotransformer tying a 600-V distribution line to a 480-V load. When it is tested as a conventional transformer, the following values are measured on the primary (480-V) side of the transformer:

Open-circuit test	Short-circuit test
$V_{OC} = 480$ V	$V_{SC} = 10.0$ V
$I_{OC} = 0.41$ A	$I_{SC} = 10.6$ A
$V_{OC} = 38$ W	$P_{SC} = 26$ W

(a) Find the per-unit equivalent circuit of this transformer when it is connected in the conventional manner. What is the efficiency of the transformer at rated conditions and unity power factor? What is the voltage regulation at those conditions?

(b) Sketch the transformer connections when it is used as a 600/480-V step-down autotransformer.

(c) What is the kilovoltampere rating of this transformer when it is used in the autotransformer connection?

(d) Answer the questions in *(a)* for the autotransformer connection.

SOLUTION

(a) The base impedance of this transformer referred to the primary side is

$$Z_{\text{base},P} = \frac{(V_P)^2}{S} = \frac{(480\text{ V})^2}{10\text{ kVA}} = 23.04\ \Omega$$

The open circuit test yields the values for the excitation branch (referred to the *primary* side):

$$\left|Y_{EX}\right| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{0.41\text{ A}}{480\text{ V}} = 0.000854\ \text{ S}$$

$$\theta = -\cos^{-1}\frac{P_{OC}}{V_{OC}\, I_{OC}} = -\cos^{-1}\frac{38\text{ W}}{(480\text{ V})(0.41\text{ A})} = -78.87°$$

$$Y_{EX} = G_C - jB_M = 0.000854\angle -78.87° = 0.000165 - j0.000838$$

$$R_C = 1/G_C = 6063\ \Omega$$

$$X_M = 1/B_M = 1193\ \Omega$$

The excitation branch elements can be expressed in per-unit as

$$R_C = \frac{6063\ \Omega}{23.04\ \Omega} = 263\text{ pu} \qquad X_M = \frac{1193\ \Omega}{23.04\ \Omega} = 51.8\text{ pu}$$

The short circuit test yields the values for the series impedances (referred to the *primary* side):

$$\left|Z_{EQ}\right| = \frac{V_{SC}}{I_{SC}} = \frac{10.0\text{ V}}{10.6\text{ A}} = 0.943\ \ \Omega$$

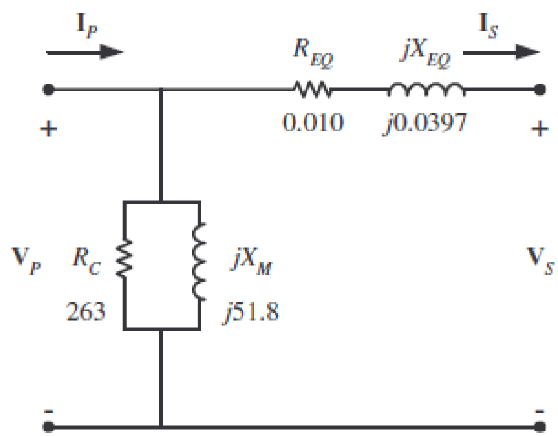
$$\theta = \cos^{-1} \frac{P_{SC}}{V_{SC}\, I_{SC}} = \cos^{-1} \frac{26 \text{ W}}{(10.0 \text{ V})(10.6 \text{ A})} = 75.8°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 0.943\angle 75.8° = 0.231 + j0.915\ \Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{0.231\ \Omega}{23.04\ \Omega} = 0.010 \text{ pu} \qquad X_{EQ} = \frac{0.915\ \Omega}{23.04\ \Omega} = 0.0397 \text{ pu}$$

The per-unit equivalent circuit is



At rated conditions and unity power factor, the input power to this transformer would be $P_{IN} = 1.0$ pu. The core losses (in resistor R_C) would be

$$P_{core} = \frac{V^2}{R_C} = \frac{(1.0)^2}{263} = 0.00380 \text{ pu}$$

The copper losses (in resistor R_{EQ}) would be

$$P_{CU} = I^2 R_{EQ} = (1.0)^2 (0.010) = 0.010 \text{ pu}$$

The output power of the transformer would be

$$P_{OUT} = P_{OUT} - P_{CU} - P_{core} = 1.0 - 0.010 - 0.0038 = 0.986$$

and the transformer efficiency would be

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{0.986}{1.0} \times 100\% = 98.6\%$$

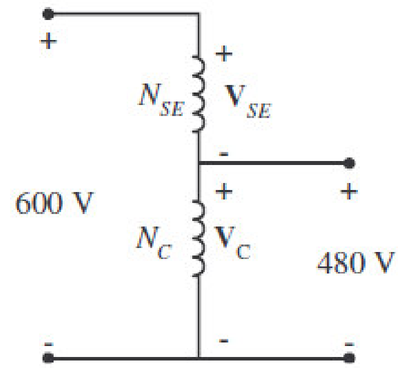
The output voltage of this transformer is

$$\mathbf{V}_{OUT} = \mathbf{V}_{IN} - \mathbf{I}Z_{EQ} = 1.0 - (1.0\angle 0°)(0.01 + j0.0397) = 0.991\angle -2.3°$$

The voltage regulation of the transformer is

$$\text{VR} = \frac{1.0 - 0.991}{0.991} \times 100\% = 0.9\%$$

(b) The autotransformer connection for 600/480 V stepdown operation is



(c) When used as an autotransformer, the kVA rating of this transformer becomes:

$$S_{IO} = \frac{N_C + N_{SE}}{N_{SE}} S_W = \frac{4+1}{1}(10\text{ kVA}) = 50\text{ kVA}$$

(d) As an autotransformer, the per-unit series impedance Z_{EQ} is decreased by the reciprocal of the power advantage, so the series impedance becomes

$$R_{EQ} = \frac{0.010}{5} = 0.002\text{ pu}$$

$$X_{EQ} = \frac{0.0397}{5} = 0.00794\text{ pu}$$

while the magnetization branch elements are basically unchanged. At rated conditions and unity power factor, the input power to this transformer would be $P_{IN} = 1.0$ pu. The core losses (in resistor R_C) would be

$$P_{core} = \frac{V^2}{R_C} = \frac{(1.0)^2}{263} = 0.00380\text{ pu}$$

The copper losses (in resistor R_{EQ}) would be

$$P_{CU} = I^2 R_{EQ} = (1.0)^2 (0.002) = 0.002\text{ pu}$$

The output power of the transformer would be

$$P_{OUT} = P_{OUT} - P_{CU} - P_{core} = 1.0 - 0.002 - 0.0038 = 0.994$$

and the transformer efficiency would be

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{0.994}{1.0} \times 100\% = 99.4\%$$

The output voltage of this transformer is

$$\mathbf{V}_{OUT} = \mathbf{V}_{IN} - \mathbf{I}Z_{EQ} = 1.0 - (1.0\angle 0°)(0.002 + j0.00794) = 0.998\angle -0.5°$$

The voltage regulation of the transformer is

$$\text{VR} = \frac{1.0 - 0.998}{0.998} \times 100\% = 0.2\%$$

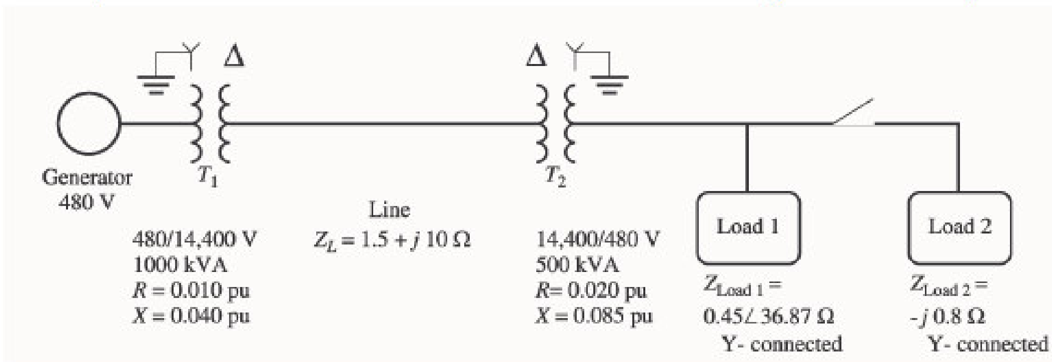
2-23. Figure P2-4 shows a power system consisting of a three-phase 480-V 60-Hz generator supplying two loads through a transmission line with a pair of transformers at either end.

(*a*) Sketch the per-phase equivalent circuit of this power system.

(*b*) With the switch opened, find the real power *P*, reactive power *Q*, and apparent power *S* supplied by the generator. What is the power factor of the generator?

(*c*) With the switch closed, find the real power *P*, reactive power *Q*, and apparent power *S* supplied by the generator. What is the power factor of the generator?

(*d*) What are the transmission losses (transformer plus transmission line losses) in this system with the switch open? With the switch closed? What is the effect of adding Load 2 to the system?



Region 1	**Region 2**	**Region 3**
$S_{\text{base1}} = 1000\text{ kVA}$	$S_{\text{base2}} = 1000\text{ kVA}$	$S_{\text{base3}} = 1000\text{ kVA}$
$V_{L,\text{base2}} = 480\text{ V}$	$V_{L,\text{base2}} = 14{,}400\text{ V}$	$V_{L,\text{base3}} = 480\text{ V}$

SOLUTION This problem can best be solved using the per-unit system of measurements. The power system can be divided into three regions by the two transformers. If the per-unit base quantities in Region 1 are chosen to be $S_{\text{base1}} = 1000\text{ kVA}$ and $V_{L,\text{base1}} = 480\text{ V}$, then the base quantities in Regions 2 and 3 will be as shown above. The base impedances of each region will be:

$$Z_{\text{base1}} = \frac{3V_{\phi 1}^{\ 2}}{S_{\text{base1}}} = \frac{3(277\text{ V})^2}{1000\text{ kVA}} = 0.238\ \Omega$$

$$Z_{\text{base2}} = \frac{3V_{\phi 2}^{\ 2}}{S_{\text{base2}}} = \frac{3(8314\text{ V})^2}{1000\text{ kVA}} = 207.4\ \Omega$$

$$Z_{\text{base3}} = \frac{3V_{\phi 3}^{\ 2}}{S_{\text{base3}}} = \frac{3(277\text{ V})^2}{1000\text{ kVA}} = 0.238\ \Omega$$

(a) To get the per-unit, per-phase equivalent circuit, we must convert each impedance in the system to per-unit on the base of the region in which it is located. The impedance of transformer T_1 is already in per-unit to the proper base, so we don't have to do anything to it:

$$R_{1,\text{pu}} = 0.010$$

$$X_{1,\text{pu}} = 0.040$$

The impedance of transformer T_2 is already in per-unit, but it is per-unit to the base of transformer T_2, so it must be converted to the base of the power system.

$$(R,\,X,\,Z)_{\text{pu on base 2}} = (R,\,X,\,Z)_{\text{pu on base 1}} \frac{(V_{\text{base 1}})^2 (S_{\text{base 2}})}{(V_{\text{base 2}})^2 (S_{\text{base 1}})} \qquad (2\text{-}60)$$

$$R_{2,\text{pu}} = 0.020 \frac{(8314\text{ V})^2 (1000\text{ kVA})}{(8314\text{ V})^2 (500\text{ kVA})} = 0.040$$

$$X_{2,\text{pu}} = 0.085 \frac{(8314\text{ V})^2 (1000\text{ kVA})}{(8314\text{ V})^2 (500\text{ kVA})} = 0.170$$

The per-unit impedance of the transmission line is

$$Z_{\text{line,pu}} = \frac{Z_{\text{line}}}{Z_{\text{base2}}} = \frac{1.5 + j10\ \Omega}{207.4\ \Omega} = 0.00723 + j0.0482$$

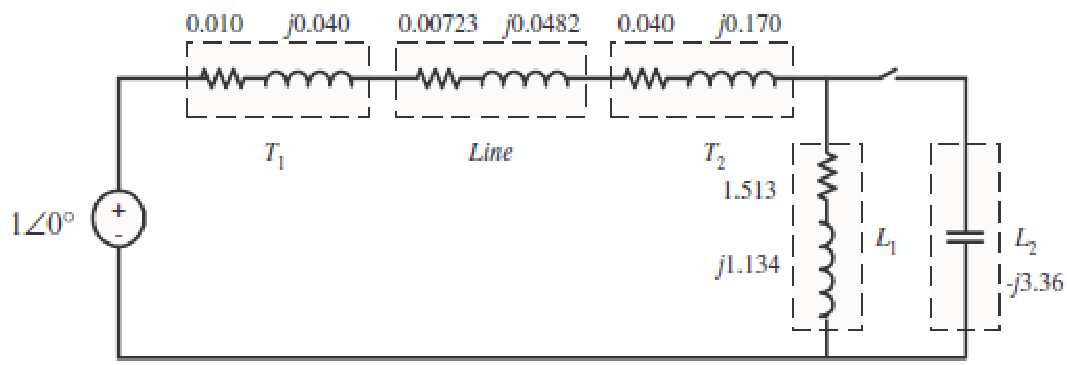
The per-unit impedance of Load 1 is

$$Z_{\text{load1,pu}} = \frac{Z_{\text{load1}}}{Z_{\text{base3}}} = \frac{0.45\angle 36.87°\ \Omega}{0.238\ \Omega} = 1.513 + j1.134$$

The per-unit impedance of Load 2 is

$$Z_{\text{load2,pu}} = \frac{Z_{\text{load2}}}{Z_{\text{base3}}} = \frac{-j0.8\ \Omega}{0.238\ \Omega} = -j3.36$$

The resulting per-unit, per-phase equivalent circuit is shown below:



(b) With the switch opened, the equivalent impedance of this circuit is

$$Z_{EQ} = 0.010 + j0.040 + 0.00723 + j0.0482 + 0.040 + j0.170 + 1.513 + j1.134$$

$$Z_{EQ} = 1.5702 + j1.3922 = 2.099\angle 41.6^\circ$$

The resulting current is

$$\mathbf{I} = \frac{1\angle 0^\circ}{2.099\angle 41.6^\circ} = 0.4765\angle -41.6^\circ$$

The load voltage under these conditions would be

$$\mathbf{V}_{\text{Load,pu}} = \mathbf{I}\, Z_{\text{Load}} = (0.4765\angle -41.6^\circ)(1.513 + j1.134) = 0.901\angle -4.7^\circ$$

$$V_{\text{Load}} = V_{\text{Load,pu}} V_{\text{base3}} = (0.901)(480\text{ V}) = 432\text{ V}$$

The power supplied to the load is

$$P_{\text{Load,pu}} = I^2 R_{\text{Load}} = (0.4765)^2 (1.513) = 0.344$$

$$P_{\text{Load}} = P_{\text{Load,pu}} S_{\text{base}} = (0.344)(1000\text{ kVA}) = 344\text{ kW}$$

The power supplied by the generator is

$$P_{G,\text{pu}} = VI\cos\theta = (1)(0.4765)\cos 41.6^\circ = 0.356$$

$$Q_{G,\text{pu}} = VI\sin\theta = (1)(0.4765)\sin 41.6^\circ = 0.316$$

$$S_{G,\text{pu}} = VI = (1)(0.4765) = 0.4765$$

$$P_G = P_{G,\text{pu}} S_{\text{base}} = (0.356)(1000\text{ kVA}) = 356\text{ kW}$$

$$Q_G = Q_{G,\text{pu}} S_{\text{base}} = (0.316)(1000\text{ kVA}) = 316\text{ kVAR}$$

$$S_G = S_{G,\text{pu}} S_{\text{base}} = (0.4765)(1000\text{ kVA}) = 476.5\text{ kVA}$$

The power factor of the generator is

$$\text{PF} = \cos 41.6^\circ = 0.748 \text{ lagging}$$

(c) With the switch closed, the equivalent impedance of this circuit is

$$Z_{EQ} = 0.010 + j0.040 + 0.00723 + j0.0482 + 0.040 + j0.170 + \frac{(1.513 + j1.134)(-j3.36)}{(1.513 + j1.134) + (-j3.36)}$$

$$Z_{EQ} = 0.010 + j0.040 + 0.00788 + j0.0525 + 0.040 + j0.170 + (2.358 + j0.109)$$

$$Z_{EQ} = 2.415 + j0.367 = 2.443\angle 8.65°$$

The resulting current is

$$\mathbf{I} = \frac{1\angle 0°}{2.443\angle 8.65°} = 0.409\angle -8.65°$$

The load voltage under these conditions would be

$$\mathbf{V}_{\text{Load,pu}} = \mathbf{I}\, Z_{\text{Load}} = (0.409\angle -8.65°)(2.358 + j0.109) = 0.966\angle -6.0°$$

$$V_{\text{Load}} = V_{\text{Load,pu}} V_{\text{base3}} = (0.966)(480 \text{ V}) = 464 \text{ V}$$

The power supplied to the two loads is the power supplied to the resistive component of the parallel combination of the two loads: 2.358 pu.

$$P_{\text{Load,pu}} = I^2 R_{\text{Load}} = (0.409)^2 (2.358) = 0.394$$

$$P_{\text{Load}} = P_{\text{Load,pu}} S_{\text{base}} = (0.394)(1000 \text{ kVA}) = 394 \text{ kW}$$

The power supplied by the generator is

$$P_{G,\text{pu}} = VI\cos\theta = (1)(0.409)\cos 6.0° = 0.407$$

$$Q_{G,\text{pu}} = VI\sin\theta = (1)(0.409)\sin 6.0° = 0.0428$$

$$S_{G,\text{pu}} = VI = (1)(0.409) = 0.409$$

$$P_G = P_{G,\text{pu}} S_{\text{base}} = (0.407)(1000 \text{ kVA}) = 407 \text{ kW}$$

$$Q_G = Q_{G,\text{pu}} S_{\text{base}} = (0.0428)(1000 \text{ kVA}) = 42.8 \text{ kVAR}$$

$$S_G = S_{G,\text{pu}} S_{\text{base}} = (0.409)(1000 \text{ kVA}) = 409 \text{ kVA}$$

The power factor of the generator is

$$\text{PF} = \cos 6.0° = 0.995 \text{ lagging}$$

(d) The transmission losses with the switch *open* are:

$$P_{\text{line,pu}} = I^2 R_{\text{line}} = (0.4765)^2 (0.00723) = 0.00164$$

$$P_{\text{line}} = P_{\text{line,pu}} S_{\text{base}} = (0.00164)(1000 \text{ kVA}) = 1.64 \text{ kW}$$

The transmission losses with the switch *closed* are:

$$P_{\text{line,pu}} = I^2 R_{\text{line}} = (0.409)^2 (0.00723) = 0.00121$$

$$P_{\text{line}} = P_{\text{line,pu}} S_{\text{base}} = (0.00121)(1000 \text{ kVA}) = 1.21 \text{ kW}$$

Load 2 improved the power factor of the system, increasing the load voltage and the total power supplied to the loads, while simultaneously decreasing the current in the transmission line and the transmission line losses. This problem is a good example of the advantages of power factor correction in power systems.