Suite TD n°1

exo 3 Calculer le flux du champ de vecteur :

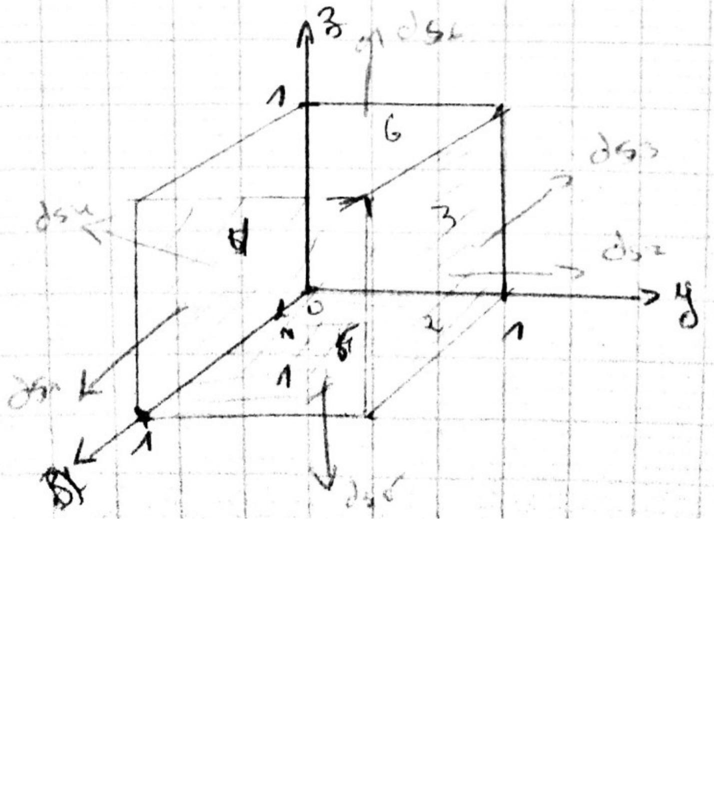
$$\vec{V}(M) = 4xz\,\vec{e_x} - y^2\,\vec{e_y} + yz\,\vec{e_z}$$

à travers la surface du cube limité.

par : $x=0$, $x=1$, $y=0$ $y=1$, $z=0$ $z=1$

définition du flux : d'un champ de vecteurs

$$\Phi = \oiint \vec{V}(M) \cdot \vec{dS} = \iint \vec{V}(M) \cdot \vec{N}\, dS$$



$\phi_{totale} = \sum_{i=1}^{6} \phi_i$

Face 1 x = 1

$V(M) = z\,\vec{e_x} - y^2\,\vec{e_y} + yz\,\vec{e_z}$

$\vec{V}\vec{M} \begin{pmatrix} 4xz \\ -y^2 \\ yz \end{pmatrix}$ $\quad d\vec{s} = ds.\vec{N_1}$ $\quad \vec{N_1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$ds = dy.dz$

$\phi_1 = \iint_{y\,z} \vec{V(M)}.\vec{N}\,.\,dy\,dz$

$\phi_1 = \int_0^1\int_0^1 (4z + 0 + 0)$

$= \int_0^1 4z\,dz.\int_0^1 dy$

$= 2z^2\Big|_0^1.\ y\Big|_0^1 \Rightarrow 2.1 = \boxed{2}$

Face 2 $\quad d\vec{s} = ds.N_2$

$\vec{V}\vec{M}\begin{pmatrix} 4xz & \vec{e_x} \\ -y^2 & \vec{e_y} \\ +yz & \vec{e_z} \end{pmatrix}$ $\quad ds = dx.dz$ $\quad \vec{N_2}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\vec{V}\vec{M}\begin{pmatrix} 4xz \\ -1 \\ z \end{pmatrix}$ $\quad ds = dx.dz$

$\phi_2 = \int_x\int_y V(\vec{M})\,\vec{N_2}\,dx\,dz$

$\phi_2 = \int_0^1\int_0^1 (0 - 1\ \ 0)\,dx\,dz$

$\phi_2 = -1\int_0^1 dx.\int_0^1 dz$

$= -1\ (1.1)$

$= -1$

Face 3 $\quad d\vec{s} = ds.N_3$

dz

$\vec{V}\vec{M}\begin{pmatrix} 4xz & \vec{e_x} \\ -y^2 & \vec{e_y} \\ +yz & \vec{e_z} \end{pmatrix}$ $\quad ds = dy.dz$ $\quad N_3\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$

$\vec{VM} - \begin{pmatrix} 0 \\ -y^2 \\ yz \end{pmatrix}$

$\phi_3 = \iint_{y\,z} V\vec{M}.N_3\,dy\,dz$

$\phi_3 = \int_0^1\int_0^1 -4xz \to \phi_3 = -4\int_0^1 dy\int_0^1 dz$

$\phi_3 = 0$

Face 4 : $\quad d\vec{S} = ds.\vec{N_4}$

$ds = dx.dz$

$N_4 : \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

$V(M)_{y=0}\begin{pmatrix} 4xz \\ 0 \\ 0 \end{pmatrix}$ $\quad \phi_4 = 5$

Face 5 $\quad d\vec{s} = ds\,\vec{N_5}$

$ds = dy\,dx$

$V(M)_{z=0} = \begin{pmatrix} 0 \\ -y^2 \\ 0 \end{pmatrix}$ $\quad N_5\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

$\phi_5 = 0$

Face 6 $\quad d\vec{s} = ds.\vec{N_5}$

$ds = dx.dy$

$V(M)_{z=1} = \begin{pmatrix} 4x \\ -y^2 \\ 0 \end{pmatrix}$ $\quad N_6\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\phi_6 = \int_0^1\int_0^1 y\,dx\,dy$

$= \int_0^1 y\,dy\int_0^1 dx = \frac{1}{2}$

$\phi = \oiint V(M)\,ds$

$= 2 - 1 + \frac{1}{2}$ $\quad \boxed{\phi = \frac{3}{2}}$

exo 4 :

On considère le champ vectoriel :

$\vec{A} = (3x^2 + 6y)\,\vec{i} - 14yz\,\vec{j} + 20\,xz^2\,\vec{k}$

a) calculer la circulation de $\vec{A}$ entre les pts :

$a\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\quad b\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

b. les segment de droites allant

$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ à $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Puis de $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ à $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ et enfin de $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ à $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

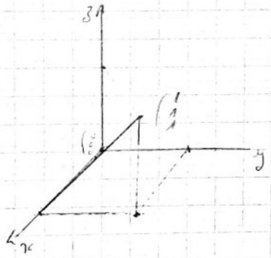
Rappel

La circulation $\mathcal{C} = \int \vec{A}\cdot\vec{dl}$

Solution

1) La circulation

$\vec{A}\begin{pmatrix} 3x^2+6y \\ -14yz \\ +20xz^2 \end{pmatrix}$



$\vec{dl}\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$

équation de droite

$a\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad b\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$x = y = z \qquad \vec{A}\begin{pmatrix} 3x^2+6x \\ -14x^2 \\ +20x^3 \end{pmatrix}$

$$\mathcal{C} = \int_0^1 \vec{A}\cdot\vec{dl}$$

$$= \int_0^1 (3x^2+6)\,dx + \int_0^1 (-14x^2)\,dx + \int_0^1 20x^3\,dx$$

$$= \cancel{\dots} = \frac{13}{3}$$

b) $\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3$

$\mathcal{C}_1 \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\vec{dl}\begin{pmatrix} dx \\ 0 \\ 0 \end{pmatrix} \quad \vec{A}\begin{pmatrix} 3x^2+ \\ 0 \\ 0 \end{pmatrix}$

$$\mathcal{C}_1 = \int_0^1 (3x^2 + \cancel{6y})\,dx = 1$$

de $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ à $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\vec{dl}\begin{pmatrix} 0 \\ dy \\ 0 \end{pmatrix} \qquad x = 1,\ z = 0$

$\vec{A}\begin{pmatrix} 3+6y \\ 0 \\ 0 \end{pmatrix}$

$$\mathcal{C}_2 = \int_0^1 (3+6y)\,\cancel{dy}^{=0} = 0$$

$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ à $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$dz \ldots$ $\vec{dl}\begin{pmatrix} 0 \\ 0 \\ dz \end{pmatrix} \qquad x = 1,\ y = 1$

$\vec{A} = \begin{pmatrix} 9 \\ -14z \\ +20z^2 \end{pmatrix}$

$$\mathcal{C}_3 = \int_0^1$$

$$\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3 = \frac{23}{3}$$

On remarque :
La circulation de champ de $\vec{A}$ dépend du chemin suivi
donc la circulation n'est pas conservative et le champ $\vec{A}$ n'est pas un champ de gradient.

correction d'exercice

$\mathrm{div}(G\vec{A}) = G\,\mathrm{div}(\vec{A}) + \vec{A}\,\vec{\mathrm{grad}}\,G$

$\mathrm{div}(\vec{A}\wedge\vec{B}) = \vec{B}\cdot(\mathrm{rot}\,\vec{A}) - \vec{A}\cdot\mathrm{rot}\,\vec{B}$

G est une fct scalaire

$G(x, y, z) \quad \vec{A}\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad G\vec{A}\begin{pmatrix} G\,A_x \\ G\,A_y \\ G\,A_z \end{pmatrix}$

$$\boxed{\mathrm{div}\,\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

$$\mathrm{div}(G\vec{A}) = \frac{\partial(G\cdot A_x)}{\partial x} + \frac{\partial(G\,A_y)}{\partial y} + \frac{\partial(G\,A_z)}{\partial z}$$

$$= \left[G\,\frac{\partial A_x}{\partial x} + A_x\,\frac{\partial G}{\partial x}\right] + \left[G\,\frac{\partial A_y}{\partial y} + A_y\,\frac{\partial G}{\partial y}\right]$$

$$+ \left[G\,\frac{\partial A_z}{\partial z} + A_z\,\frac{\partial G}{\partial z}\right]$$

$$= G\left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right] + A_x\,\frac{\partial G}{\partial x} + A_y\,\frac{\partial G}{\partial y} + A_z\,\frac{\partial G}{\partial z}$$

$$= G\,\mathrm{div}(\vec{A}) + \vec{A}\cdot\vec{\mathrm{grad}}(G)$$

2) $\vec{A}\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \quad \vec{B}\begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$

$$\vec{A}\wedge\vec{B} = \begin{vmatrix} i & a_x & b_x \\ j & a_y & b_y \\ k & a_z & b_z \end{vmatrix} = \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

$$\mathrm{div}(\vec{A}\wedge\vec{B}) = \frac{\delta(a_y b_z - a_z b_y)}{\delta x} + \frac{\delta(a_z b_x - a_x b_z)}{\delta y}$$

$$+ \frac{\delta(a_x b_y - a_y b_x)}{\delta z}$$

$$= \frac{\delta(a_y)}{\delta x} b_z + \frac{\delta(b_z)}{\delta x} a_y - \left[\frac{\delta a_z}{\delta x} b_y + \frac{\delta b_y}{\delta x} a_z\right]$$

$$+ \frac{\delta(a_z)}{\delta y} b_x + \frac{\delta(b_x)}{\delta y} a_z - \left[\frac{\delta(a_x)}{\delta y} b_z + \frac{\delta(b_z)}{\delta y} a_x\right]$$

$$+ \frac{\delta(a_x)}{\delta z} b_y + \frac{\delta(b_y)}{\delta z} a_x - \left[\frac{\delta(a_y)}{\delta z} b_x + \frac{\delta(b_x)}{\delta z} a_y\right]$$

$$= b_z \cdot ($$

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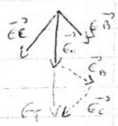
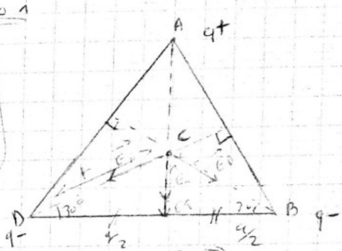
Rappel Exo 1





$$\vec{E_T} = \vec{E_A} + \vec{E_B} + \vec{E_D}$$

l'intensité de E_T :

$$E = q \cdot \frac{1}{4\pi\varepsilon_0 r^2}$$

1^ère Méth

$$\frac{a}{\sin 120} = \frac{CD}{\sin 30} = \frac{CB}{\sin 30}$$

$$CD = \frac{a}{\sin 120} \sin 30$$

2^ème Méth

calcul de r

$$\cos 30° = \frac{a/2}{r}$$

$$r = \frac{a/2}{\cos 30} \Rightarrow r = \frac{a}{\sqrt{3}}$$

http://eltblida.blogspot.com/p/theorie-du-champ.html