Exercice 1

assume(real) = real

$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$; $\vec{V}_2 = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$; $\vec{V}_3 = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$\vec{A} = \vec{V}_1 + \vec{V}_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$; $\vec{B} = \vec{V}_3 - \vec{V}_2 = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$; $\vec{C} = \vec{V}_1 - \vec{V}_2 + 2\vec{V}_3 = \begin{pmatrix} 14 \\ -2 \end{pmatrix}$

Exercice 2

$\vec{V}_1 = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$; $\vec{V}_2 = \begin{pmatrix} 2 \\ -4 \\ -3 \end{pmatrix}$; $\vec{V}_3 = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$

a. Module $\left\|\vec{A}\right\| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\left\|\vec{V}_1\right\| = \sqrt{14} = 3.74$; $\left\|\vec{V}_2\right\|_2 = \sqrt{29} = 5.39$; $\left\|\vec{V}_3\right\|_2 = 3$

$\vec{A} = \vec{V}_1 + \vec{V}_2 + \vec{V}_3$, $\vec{A} = \begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix}$; $\left\|\vec{A}\right\|_2 = \sqrt{32} = 5.66$

$\vec{B} = 2\vec{V}_1 - 3\vec{V}_2 - 5\vec{V}_3$; $\vec{B} = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$; $\left\|\vec{B}\right\|_2 = \sqrt{30} = 5.48$

b. Angles : On utilise $\vec{V}_1 \cdot \vec{V}_2 = \left\|\vec{V}_1\right\| \left\|\vec{V}_2\right\| \cos(\alpha) \Rightarrow \cos(\alpha) = \frac{\vec{V}_1 \cdot \vec{V}_2}{\left\|\vec{V}_1\right\| \left\|\vec{V}_2\right\|}$

$\vec{V}_1 \cdot \vec{V}_2 = 11$; $\cos(\alpha) = \frac{11}{\sqrt{14}\sqrt{29}} = 0.545\,92 \Rightarrow \alpha = \pm 56.91°$

$\vec{V}_1 \cdot \vec{V}_3 = -5$; $\cos(\alpha) = \frac{-5}{3\sqrt{14}} = -0.445\,44 \Rightarrow \alpha = \pm 63.55°$

$\vec{V}_2 \cdot \vec{V}_3 = -16$; $\cos(\alpha) = \frac{-16}{3\sqrt{29}} = -0.990\,38 \Rightarrow \alpha = \pm 7.96°$

Exercice 3

On utilise $\vec{V}_1 \cdot \vec{V}_2 = 0$

$\vec{V}_1 = \begin{pmatrix} 2 \\ a \\ 1 \end{pmatrix}$; $\vec{V}_2 = \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}$; Donc $\vec{V}_1 \cdot \vec{V}_2 == 6 - 2a = 0 \Rightarrow a = 3$

Exercice 4

Evident

Exercice 5

Méthode 1

$\vec{V}_1 \times \vec{V}_2 = \left(2\vec{i} + 2\vec{j}\right) \times \left(-4\vec{i}\right) = -8\left(\vec{i} \times \vec{i}\right) - 8\left(\vec{j} \times \vec{i}\right) = \vec{0} + 8\vec{k}$

$\vec{V}_1 \times \vec{V}_3 = \left(2\vec{i} + 2\vec{j}\right) \times \left(4\vec{i} - 2\vec{j}\right) = -4\left(\vec{i} \times \vec{j}\right) + 8\left(\vec{j} \times \vec{i}\right) = -12\vec{k}$

$\vec{V}_2 \times \vec{V}_3 = \left(-4\vec{i}\right) \times \left(4\vec{i} - 2\vec{j}\right) = 8\left(\vec{i} \times \vec{j}\right) = 8\vec{k}$

Méthode 2

$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$; $\vec{V}_2 = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix}$; $\vec{V}_3 = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$

$$\vec{V}_1 \times \vec{V}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 0 \\ -4 & 0 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 0 \\ -4 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 2 \\ -4 & 0 \end{vmatrix} = 8\vec{k} = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$$

$$\vec{V}_1 \times \vec{V}_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 0 \\ 4 & -2 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & 0 \\ -2 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 0 \\ 4 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 2 \\ 4 & -2 \end{vmatrix} = -12\vec{k} = \begin{pmatrix} 0 \\ 0 \\ -12 \end{pmatrix}$$

$$\vec{V}_2 \times \vec{V}_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 0 & 0 \\ 4 & -2 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & 0 \\ -2 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 0 \\ 4 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} -4 & 0 \\ 4 & -2 \end{vmatrix} = 8\vec{k} = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$$

Exercice 6

$\vec{V}_1 = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}; \vec{V}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}; \ \vec{V}_3 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$\left(\vec{V}_1 \times \vec{V}_2\right) \cdot \vec{V}_3 = -5$; $\left(\vec{V}_1 \times \vec{V}_2\right) \times \vec{V}_3 = \begin{pmatrix} 24 \\ 7 \\ -5 \end{pmatrix}$; $\vec{V}_1 \times \left(\vec{V}_2 \times \vec{V}_3\right) = \begin{pmatrix} 15 \\ 15 \\ -15 \end{pmatrix}$; $\vec{V}_1 \times \left(\vec{V}_2 \cdot \vec{V}_3\right) = Vec \times (scal) =$ impossible; $\vec{V}_1 \cdot \left(\vec{V}_2 \times \vec{V}_3\right) = -5$

$\left(\vec{V}_1 \cdot \vec{V}_2\right) \cdot \vec{V}_3 = (scal) \cdot vect =$ impossible (mais $\left(\vec{V}_1 \cdot \vec{V}_2\right) \vec{V}_3 = \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix}$);

$\left(\vec{V}_1 + \vec{V}_2\right) \times \left(\vec{V}_1 - \vec{V}_2\right) = -\left(\vec{V}_1 \times \vec{V}_2\right) + \left(\vec{V}_2 \times \vec{V}_1\right) = -2\left(\vec{V}_1 \times \vec{V}_2\right) = \begin{pmatrix} 2 \\ -14 \\ -10 \end{pmatrix}$

Exercice 7

assume(real) = real

$\vec{V}_1 = \begin{pmatrix} \sin t \\ \cos t \\ t \end{pmatrix}; \vec{V}_2 = \begin{pmatrix} \exp(-t) \\ 2\cos(3t) \\ 2\sin(3t) \end{pmatrix} \Rightarrow \frac{d\vec{V}_1}{dt} = \begin{pmatrix} \cos t \\ -\sin t \\ 1 \end{pmatrix}; \ \frac{d\vec{V}_2}{dt} = \begin{pmatrix} -e^{-t} \\ -6\sin 3t \\ 6\cos 3t \end{pmatrix}$

$\left\| \frac{d\vec{V}_1}{dt} \right\| = 0$; $\left\| \frac{d\vec{V}_2}{dt} \right\| = \sqrt{36 + e^{-2t}}$