

Exercice 1 (6 Points)

$$V_1 = \frac{1}{4} (V_2 + 0 + 30 - 20) \text{ et } V_2 = \frac{1}{4} (V_1 + 20 + 0 + 30)$$

$$\text{Soit } V_1 = \frac{1}{4} (V_2 + 10) \text{ et } V_2 = \frac{1}{4} (V_1 + 50)$$

Itération	1	2	3	4	5
V_1	2.500	① 5.781	① 5.986	① 5.999	① 6.000
V_2	13.125	13.945	13.996	13.999	14.000

Exercice 2 (4 Points)

$$a) \frac{dV}{dx} \Big|_{x=x_0} \approx \frac{V(x_0+dx) - V(x_0-dx)}{2dx} \text{ et } \frac{d^2V}{dx^2} \Big|_{x=x_0} \approx \frac{V(x_0+dx) - 2V(x_0) + V(x_0-dx)}{(dx)^2}$$

avec $x_0 = 0.15$ et $dx = 0.05$ soit :

$$\frac{dV}{dx} \Big|_{x=0.15} = 10.1170$$

$$\frac{d^2V}{dx^2} \Big|_{x=0.15} = 1.5600$$

b) $N = 10 \sinh(x) \rightarrow$

$$V' = 10 \cosh(x) \text{ et } V'' = 10 \sinh(x) \rightarrow V'_x(0.15) = 10.1127 \text{ et } V''_x(0.15) = 1.5056$$

Exercice 3 (10 Points)

$$P_1 = (y_2 - y_3), P_2 = (y_3 - y_1), P_3 = (y_1 - y_2); Q_1 = (x_3 - x_2), Q_2 = (x_1 - x_3), Q_3 = (x_2 - x_1)$$

$$A = 1/2 (P_2 Q_3 - P_3 Q_2); C_{ij}^{(e)} = 1/4A [P_i P_j + Q_i Q_j]$$

élément 1

Nœud	Coordonnée
1	(6, 0)
2	(3, 6)
3	(0, 3)

$$P_1 = 3, P_2 = 3, P_3 = -6$$

$$Q_1 = -3, Q_2 = 6, Q_3 = -3$$

$$A = 13.5$$

$$C^{(1)}$$

$$C^{(1)} = \begin{bmatrix} 0.3333 & -0.1667 & -0.1667 \\ -0.1667 & 0.8333 & -0.6667 \\ -0.1667 & -0.6667 & 0.8333 \end{bmatrix}$$

élément 2

Nœud	Coordonnée
1	(6, 0)
2	(6, 6)
3	(3, 6)

$$P_1 = 0, P_2 = 6, P_3 = -6$$

$$Q_1 = -3, Q_2 = 3, Q_3 = 0$$

$$A = 9$$

$$C^{(2)}$$

$$C^{(2)} = \begin{bmatrix} 0.25 & -0.25 & 0.00 \\ -0.25 & 1.25 & -1.00 \\ 0.00 & -1.00 & 1.00 \end{bmatrix}$$

La matrice globale C

$$C_{11} = C_{11}^{(1)} + C_{11}^{(2)} = 0.5833$$

$$C_{12} = C_{12}^{(2)} = -0.25 = C_{21}$$

$$C_{13} = C_{12}^{(1)} + C_{13}^{(2)} = -0.1667 = C_{31}$$

$$C_{14} = C_{13}^{(1)} = -0.1667 = C_{41}$$

$$C_{22} = C_{22}^{(2)} = 1.25$$

$$C_{23} = C_{23}^{(2)} = -1.00 = C_{32}$$

$$C_{24} = 0 = C_{42}$$

$$C_{33} = C_{22}^{(1)} + C_{33}^{(2)} = +1.8333$$

$$C_{34} = C_{23}^{(1)} = -0.6667 = C_{43}$$

$$C_{44} = C_{33}^{(1)} = 0.8333$$

$$C = \begin{bmatrix} 0.5833 & -0.25 & -0.1667 & -0.1667 \\ -0.25 & 1.25 & -1.00 & 0.00 \\ -0.1667 & -1.00 & 1.8333 & -0.6667 \\ -0.1667 & 0.00 & -0.6667 & 0.8333 \end{bmatrix}$$